1. Find the average rate of change from $x = -1$ to $x = 2$ for each of the functions below.

   a. $a(x) = 2x + 3$
   b. $b(x) = x^2 - 1$
   c. $c(x) = 2^x + 1$

   d. Which function has the greatest average rate of change over the interval $[-1, 2]$?

   \[
   \begin{align*}
   a(-1) &= 2(-1) + 3 = 1 \\
   a(2) &= 4 + 3 = 7 \\
   M &= \frac{Y_2 - Y_1}{x_2 - x_1} = \frac{7 - 1}{2 - (-1)} = \frac{6}{3} = 2
   \end{align*}
   \]

   \[
   \begin{align*}
   b(-1) &= (-1)^2 - 1 = 0 \\
   b(2) &= 4 - 1 = 3 \\
   M &= \frac{Y_2 - Y_1}{x_2 - x_1} = \frac{3 - 0}{2 - (-1)} = \frac{3}{3} = 1
   \end{align*}
   \]

   \[
   \begin{align*}
   c(-1) &= 2^{-1} + 1 = 1.5 \\
   c(2) &= 4 + 1 = 5 \\
   M &= \frac{Y_2 - Y_1}{x_2 - x_1} = \frac{5 - 1.5}{2 - (-1)} = \frac{3.5}{3} = 1.1\bar{6}
   \end{align*}
   \]

2. Find the average rate of change on the interval $[2, 5]$ for each of the functions below.

   a. $a(x) = 2x + 1$
   b. $b(x) = x^2 + 2$
   c. $c(x) = 2^x - 1$

   d. Which function has the greatest average rate of change over the interval $x = 2$ to $x = 5$?

   \[
   \begin{align*}
   a(2) &= 4 + 1 = 5 \\
   a(5) &= 10 + 1 = 11 \\
   M &= \frac{Y_2 - Y_1}{x_2 - x_1} = \frac{11 - 5}{5 - 2} = \frac{6}{3} = 2
   \end{align*}
   \]

   \[
   \begin{align*}
   b(2) &= 4 + 2 = 6 \\
   b(5) &= 25 + 2 = 27 \\
   M &= \frac{Y_2 - Y_1}{x_2 - x_1} = \frac{27 - 6}{5 - 2} = \frac{21}{3} = 7
   \end{align*}
   \]

   \[
   \begin{align*}
   c(2) &= 4 - 1 = 3 \\
   c(5) &= 32 - 1 = 31 \\
   M &= \frac{Y_2 - Y_1}{x_2 - x_1} = \frac{31 - 3}{5 - 2} = \frac{28}{3} = 9\frac{1}{3}
   \end{align*}
   \]

3. In general as $x \to \infty$, which function eventually grows at the fastest rate?

   a. $a(x) = 2x$
   b. $b(x) = x^2$
   c. $c(x) = 2^x$

   \[
   \begin{align*}
   \text{Plot1} & \quad \text{Plot2} & \quad \text{Plot3}
   \end{align*}
   \]
4. Find the average rate of change from \( x = -1 \) to \( x = 2 \) for each of the continuous functions below based on the partial set of values provided.

\[
\begin{array}{c|c|c|c}
\hline
x & a(x) & b(x) & c(x) \\
\hline
-1 & -3 & 1 & -2 \\
0 & -2 & 3 & -1 \\
1 & 1 & 7 & 1 \\
2 & 6 & 9 & 5 \\
3 & 13 & 13 & 13 \\
\hline
\end{array}
\]

\[M_a = \frac{Y_2 - Y_1}{x_2 - x_1} = \frac{6 - (-3)}{2 - (-1)} = \frac{9}{3} = 3 \]

\[M_b = \frac{Y_2 - Y_1}{x_2 - x_1} = \frac{7 - 1}{2 - (-1)} = \frac{6}{3} = 2 \]

\[M_c = \frac{Y_2 - Y_1}{x_2 - x_1} = \frac{5 - (-2)}{2 - (-1)} = \frac{7}{3} = 2.3 \]

\[\text{d. Which function has the greatest average rate of change over the interval } [-1, 2]? \]

\[a(x)\]

5. Consider the table below that shows a partial set of values of two continuous functions. Based on any interval of \( x \) provided in the table which function always has a larger average rate of change?

\[
\begin{array}{c|c|c}
\hline
x & f(x) & g(x) \\
\hline
-1 & -2 & -4 \\
0 & 0 & 0 \\
1 & 3 & 8 \\
2 & 7 & 24 \\
\hline
\end{array}
\]

\[\text{g(x)}\]

6. Find the average rate of change from \( x = 1 \) to \( x = 3 \) for each of the functions graphed below.

\[\text{a.}\]

\[\text{b.}\]

\[\text{c.}\]

\[\text{d. Find an interval of } x \text{ over which all three graphed functions above have the same average rate of change.}\]

\[f(x)\]