Storm the Castle
Quadratic Regression

Goal: Today you will make and test a carnival game. The game is “Storm the Castle.” You will make a popsicle stick catapult and test it to make it reliable with small variability. You will then have the opportunity to catapult a marshmallow at a castle. If you hit the castle, you win the prize.

Instructions for making a popsicle stick catapult are modified from the following website: http://www.stormthecastle.com/catapult/popsiclestick-catapult.htm

Materials for one catapult: 32 popsicle sticks, masking tape, one long rubber band (or 2 small linked together), scissors

You can see this assembly process at the following website: http://www.stormthecastle.com/catapult/teeny-tiny-catapult.htm

This catapult project is an easy and fun project. It takes about thirty minutes. It also shows you a fundamental concept of engineering - the strength of the triangle.

A little About the Triangle - The thing that makes this catapult so easy to build yet so strong (You can use quite a potent rubber band on this catapult and fire your projectiles a long way) is the use of the triangle which is the strongest geometric shape and is used in everywhere in architecture. It is what is used to make geodesic domes which is what Epcot center at Walt Disney World is made out of. The geodesic dome is a series of triangles that are formed together so the triangles are now in three dimensional shapes like this catapult.

Step 1: You will need to construct 3 equilateral triangles, so 3 people in your group should construct the following while the remaining person reads the instructions:
Here is the basic process for taping sticks together. Tear off 3” of tape. Lay out your tape with the sticky side up then place two sticks end to end perpendicular to the tape. But just press the sticks down firmly on the tape, don't wrap it up yet. Place a third stick on top of these two sticks, right in the middle. Now you can wrap the tape around all three. Do at least two full revolutions of the tape.

Step 2:
Each person should make three of these. This is the basis for your first triangle.

Step 3: Now, as in the photo shown above, place these three popsicle stick assemblies end to end (with the 3rd stick facing up) and then tape them together. Use at least three wraps of tape at the joints to make it nice and strong.

Now fold the length of sticks into a triangle (where the 3rd stick is on the inside of the triangle) and tape the ends together. You now have your first triangle. Do this process two more times so you have three triangles.
Step 4:
Now hold two of the triangles together side by side and tape them only on one side (in the middle). The red arrow shows the side that I have taped together.

Step 5:
Now lay one triangle down on the table and splitting the legs on the two you taped together slide them right over the one on the table.

Step 6:
Here is the completed triangle; now all that remains is to tape all the shapes together.

Step 7:
Now let's build the arm of the catapult.

We use five popsicle sticks and this picture shows you how to arrange them. Put three end to end then put two right on top of them. The picture shows the two top sticks like this just so you see the five sticks, you place the two sticks right on top of the three then tape them.

Here is my catapult arm completed.

Step 8:
Now make the catapult cup out of paper. Here is a template of how to cut it out. All dimensions are either one inch or one half inch. Draw this out on paper and cut it out. Fold it along the dotted lines to form a cup and tape it into shape. Then tape it to the arm of your catapult. Don't tape it to the end of the arm, affix it about an inch from the end so you have room to press down the arm with your finger.
Here is the paper cup taped onto the end of the catapult. Use plenty of tape and make it nice and strong.

Now wrap one end of the rubber band around the catapult arm just under the cup and tape it firmly in place.

Using a couple of pieces of tape connect the other end of the catapult arm to the place where the three triangles meet. (Note: the catapult arm should go through the inside of the triangles) This is a bit tricky to do but do your best and try to secure the catapult arm firmly. But it has to move freely. Think of the tape as being a kind of a hinge.

The only thing left to do now is to wrap the other end of the rubber band around the top of the catapult and tape it firmly into place. If you do not have a large rubber band you can chain a group of small ones together then use them. Experiment with the rubber bands and get a power set up that you like.

**Testing the Catapult:**

I. Average Distance Travelled

   a) Put a marshmallow in your catapult cup. Shoot the marshmallow 5 times and record its distance from the catapult when it **first** hits the ground in the table below:

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<th>distance</th>
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   b) What is the average distance from the catapult that the marshmallow travelled?

   c) Compute the standard deviation _________. Check your solution by hand for homework:

   \[
   \sigma = \sqrt{\frac{1}{N} \sum_{i=1}^{N} (x_i - \bar{x})^2}
   \]

   d) Are there any outliers in your data (values that are more than 2 standard deviations away from the mean)? Show justification below:
II. Reducing Variability

a) Write down something that you plan to do to reduce the variability.

b) Make that adjustment and shoot the marshmallow 5 more times. Record the data below:

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c) What is the average distance from the catapult that the marshmallow travelled?___________

d) Compute the standard deviation. ____________ . Check your solution by hand for homework:

e) Did your method reduce the variability? Explain.

f) Were there any marshmallows that travelled unusual distances? If so, remove that value and recalculate the standard deviation. How does this standard deviation compare to the one in part d?

III. Finding the best fit model

a) As you watched the marshmallow fly in the air, what kind of path did the marshmallow make as you shot it from the catapult?

b) Now, we will collect data to find the height of the marshmallow at certain distances from the catapult. Pick a distance from the catapult, and as the marshmallow passes you by estimate the height. Record this data in the table below. Repeat the process at different distances from the catapult.

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<thead>
<tr>
<th>Distance from catapult</th>
<th>Height of marshmallow</th>
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c) Make a scatterplot of your data on the grid:

d) Enter the data into our TI-84 lists by choosing “Stat, edit.” Put “distance from catapult” in L1 and “height of marshmallow” in L2.

On your calculator, choose “Stat, calc, QuadReg (#5), enter, enter” to perform quadratic regression on your calculator to come up with a model for your data:_________________________________
Choose the “Y=” button at the top left of your calculator and type the quadratic regression equation.

Press “2nd, Y=” (which is the stat plot menu), hit enter to select Plot 1, then select the scatterplot option under “Type”. Make sure that XList is L1 and YList is L2. See picture on the right.

Now press zoom #9 to see your scatterplot and quadratic model. Discuss whether you have a good fit or not.

IV. Based on the model graphed on your calculator, answer the following questions. Use either the trace feature of your calculator or go to the menu under “2nd, Calc” to help you answer the following.

a) What was the initial height of the marshmallow (at distance = 0)?_______

b) On average, what was the maximum height of the marshmallow?_________

c) On average, how high off the ground was the marshmallow when it was 12 inches away from the catapult?________

d) On average, at what distance(s) from the catapult was the marshmallow 1 foot off of the ground?_________

V. Play the game…storm the castle! Place both the catapult and the castle on the ground or on the same table. You are now going to play the game.

a) Measure the height of the castle.______________________

b) Use this height and your calculator to determine how far your catapult needs to be in order for the marshmallow to hit the top of the castle on the way down.

c) How far away does the catapult need to be in order to hit the bottom of the castle?______________________

d) You have 3 chances to hit the castle. Use parts b and c to place the castle where it needs to be. Now, play the game. Did you hit the castle?________ If not, what factors could have caused the variability?

Optional:
The marshmallow must travel through a hula hoop that’s at least 3 feet from the catapult before it hits the castle. Measure the diameter of the hula hoop. _______________ How far away from the castle must it be in order for the marshmallow to make it through?__________________