Section 4-3 Rational & Irrational

**Rational Number**: A rational number is one that *can* represented as a ratio of \( \frac{p}{q} \), such that p and q are both integers and \( q \neq 0 \). All rational numbers *can* be expressed as a terminating or repeating decimal. (Examples: \(-0.5, 0, 7, \frac{3}{2}, 0.2\overline{6}\))

**Irrational Number**: An irrational number is one that *cannot* represented as a ratio of \( \frac{p}{q} \), such that p and q are both integers and \( q \neq 0 \). Irrational numbers *cannot* be expressed as a terminating or repeating decimal. (Examples: \( \sqrt{3}, \pi, \frac{\sqrt{5}}{2}, e \))

Irrational numbers are difficult to comprehend because they cannot be expressed easily. Consider creating a physical representation of the \( \sqrt{2} \). Create a right triangle with each leg being exactly 1 meter. The hypotenuse should then be the length of \( \sqrt{2} \) meters.

What is interesting is, no matter how precise the ruler is, you can never measure the exact length of the hypotenuse using a metric scale. The hypotenuse will ALWAYS fall between any two lines of a metric division. This bothered early Greeks specifically the Pythagoreans. They thought it was illogical or crazy (i.e. irrational) that it was possible to draw a line of a length that could *NEVER* be measured precisely using a scale that was some integer division of the original measures. They even hid the fact that they may have known this as they believed it to be an imperfection of mathematics.

In the example above the hypotenuse is approximately 1.41421356237095048801688724209698.... meters. It can *NEVER* be written precisely as a decimal which may seem a little IRRATIONAL. The decimal description goes on forever without a repeating pattern.
A set of numbers is said to be closed under an operation if any two numbers from the original set are combined under the operation and the solution is always in the same set as the original numbers.

For example, the sum of any two even numbers always results in an even number. So, the set of even numbers is closed under addition.

For example, the sum of any two odd numbers always results in an even number. So, the set of even numbers is NOT closed under addition.

1. Is the set of **Integers** closed under **addition**?
   
   Yes
   
   No

2. Is the set of **Integers** closed under **subtraction**?
   
   Yes
   
   No

3. Is the set of **Integers** closed under **multiplication**?
   
   Yes
   
   No

4. Is the set of **Integers** closed under **division**?
   
   Yes
   
   No

5. Is the set of **Rational Numbers** closed under **addition**?
   
   Yes
   
   No

6. Is the set of **Irrational Numbers** closed under **addition**?
   
   Yes
   
   No

7. Is the set of **Rational Numbers** closed under **multiplication**?
   
   Yes
   
   No

8. Is the set of **Irrational Numbers** closed under **multiplication**?
   
   Yes
   
   No

9. Is the set of **Even Numbers** closed under **division**?
   
   Yes
   
   No

10. Is the set of **Odd Numbers** closed under **multiplication**?
    
    Yes
    
    No
Tell whether you think the following numbers are Rational or Irrational.

11. \(\sqrt{8}\)
   
   \[\sqrt{8} = 2\sqrt{2} \approx 2.828427125\ldots\]
   
   Circle One: Rational  Irrational

12. \(\sqrt{2} + \sqrt{49}\)
   
   \[\sqrt{2} + 7 = \sqrt{2} + 7 = \sqrt{9} = 3\]
   
   Circle One: Rational  Irrational

13. \(2\sqrt{27} - \sqrt{3} - \sqrt{75}\)
   
   \[2\cdot3\sqrt{3} - \sqrt{3} - 5\sqrt{3} = 0\]
   
   Circle One: Rational  Irrational

14. \(\pi\)
   
   \[\pi \approx 3.141592654\ldots\]
   
   Circle One: Rational  Irrational

15. \(\sqrt{12} \cdot \sqrt{3}\)
   
   \[= \sqrt{36} = 6\]
   
   Circle One: Rational  Irrational

16. \(e^2\)
   
   \[e^2 \approx 7.389056099\ldots\]
   
   Circle One: Rational  Irrational

17. \(\sqrt{64} = 4\)
   
   Circle One: Rational  Irrational

18. \(\sqrt{24}\)
   
   \[2\sqrt{3} \approx 2.886751345\ldots\]
   
   Circle One: Rational  Irrational

19. \(3.\overline{23131}\)
   
   \[3.231313131313131313131\ldots \approx \frac{31333}{990}\]
   
   Circle One: Rational  Irrational

20. \(3.12111211121112\ldots\)
   
   Pattern exists but not a rational one
   
   Circle One: Rational  Irrational

21. \(\varphi = \frac{1 + \sqrt{5}}{2} \approx 1.618034\ldots\)
   
   Golden ratio is irrational (Ironic that it is called a ratio)
   
   Circle One: Rational  Irrational

22. \(81^{\frac{3}{4}} = \frac{1}{81^{\frac{1}{4}}} = (\sqrt[4]{81})^3 = \frac{1}{(81^{\frac{1}{4}})^3} = \frac{1}{(9)^3} = \frac{1}{27}\)
   
   Circle One: Rational  Irrational