### EASY

\[ x^2 + 10x - 6 = 0 \]

Move constant to the other side

\[ x^2 + 10x = 6 \]

\[ x^2 + 10x + 25 = 31 \]

Easily Factors

\[ (x + 5)(x + 5) = 31 \]

Can be re-written

\[ (x + 5)^2 = 31 \]

Take the square root of both sides

\[ \sqrt{(x + 5)^2} = \sqrt{31} \]

Don’t forget \( \pm \).

Isolate \( x \)

\[ x + 5 = \pm \sqrt{31} \]

\[ x = -5 \pm \sqrt{31} \]

\( \text{OR} \)

\[ x \approx 0.5678 \text{ or } -10.5678 \]

### MEDIUM

\[ 3x^2 - 15x + 12 = 0 \]

Move constant to the other side

\[ 3x^2 - 15x = -9 \]

Divide both sides by the leading coefficients

\[ \frac{3x^2 - 15x}{3} = \frac{-9}{3} \]

\[ x^2 - 5x = -3 \]

\[ x^2 + 10x + \frac{25}{4} = -\frac{3}{1} + \frac{25}{4} \]

\[ \left( x - \frac{5}{2} \right)^2 = \frac{13}{4} \]

\[ x - \frac{5}{2} = \pm \frac{\sqrt{13}}{2} \]

\[ x = \frac{5 \pm \sqrt{13}}{2} \]

\[ x \approx 0.6972 \text{ or } -4.3028 \]
Solve the following by completing the square.

1. \( x^2 + 10x - 19 = 0 \)  
2. \( x^2 - 12x + 2 = 0 \)

3. \( x^2 + 15x - 8 = 0 \)  
4. \( x^2 - 7x - 4 = 0 \)

5. \( 2x^2 - 8x - 14 = 0 \)  
6. \( 3x^2 - 18x - 5 = 0 \)
Solve the following by completing the square.

7. \(2x^2 - 10x - 3 = 0\)  
8. \(3x^2 - 21x - 5 = 0\)

9. \(2x^2 - 5x - 8 = 0\)  
10. \(3x^2 - 10x - 7 = 0\)
Solve the following by completing the square. Derive the quadratic formula.

\[ a \cdot x^2 + b \cdot x + c = 0 \]
The Quadratic Formula

Solve the following by quadratic formula

\[ x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \]

1. \( x^2 + 10x - 19 = 0 \)
2. \( x^2 - 12x + 2 = 0 \)

3. \( x^2 + 15x - 8 = 0 \)
4. \( x^2 - 7x - 4 = 0 \)

5. \( 2x^2 - 8x - 14 = 0 \)
6. \( 3x^2 - 18x - 5 = 0 \)

7. \( 2x^2 - 10x - 3 = 0 \)
8. \( 3x^2 - 21x - 5 = 0 \)

M. Winking (Section 5-4)
The **discriminant** determines the type of solution **Rational** or **Irrational** and **Real** or **Imaginary**.

\[
x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}
\]

\[
b^2 - 4ac = \text{discriminant}
\]

### Positive Perfect Square
- **Examples**: 9, 25, 49
- If the discriminate is a positive perfect square then the square root can be completely eliminated when the radical is simplified and no radical will be left over. So the solution can be written as **2 REAL** and **RATIONAL** number.

### Positive Non Square
- **Examples**: 7, 12, 22
- If the discriminate is positive but NOT a perfect square then there will still be **2 REAL** solutions but the solutions will be **IRRATIONAL** since the radical cannot be completely eliminated.

### Zero
- **Examples**: 0
- If the discriminate is zero then there will be just **1 REAL** **RATIONAL** solution because adding or subtracting 0 is equivalent.

### Negative
- **Examples**: −3, −9, −12
- If the discriminate is negative then there will be **2 IMAGINARY** solutions (there will be an \(i\) in the solution).

Describe the nature of the roots using the discriminant.

1. \(2x^2 - 8x - 14 = 0\)  
2. \(3x^2 + 2x + 8 = 0\)

3. \(3x^2 - 15x + 12 = 0\)  
4. \(3x^2 - 10x - 7 = 0\)

5. \(8x^2 - 24x + 18 = 0\)  
6. \(x^2 - 6x + 13 = 0\)

7. \(12x^2 - 34x + 24 = 0\)

8. Give a possible value for ‘\(a\)’ such that the solution of the quadratic would have **2 real rational solutions**.

\[
a x^2 - 5x - 2 = 0
\]
1. Find the value of $x$ that would make the diagram below accurate.

![Diagram with $x+2$, $x$, and $x+4$ labels]

2. A golf ball is hit with an initial vertical velocity of 80 fps

$$h = -16t^2 + 80t$$

a. How high is the ball after 2 seconds?

b. How many seconds would it take the ball to hit the ground (the height would be $h=0$)?

c. When will the ball reach 48 feet?

d. When does the ball reach a maximum height?

e. What is the average height from $t = 1$ to $t = 2$ seconds?

f. For what values of $t$ is the domain appropriate?
3. A baseball hit with an initial vertical velocity of 121 fps and the ball was struck 1 foot above ground.

\[ h = -16t^2 + 121t + 1 \]

a. How high is the ball after 2 seconds?

b. How many seconds would it take the ball to hit the ground (the height would be \( h=0 \))? 

c. When will the ball reach 30 feet?

d. When does the ball reach a maximum height?

e. What is the average vertical velocity from \( t = 0.2 \) to \( t = 0.9 \) seconds?

4. A person at a framing store is making a frame mat to go around a picture. The mat is a uniform 2 inches around on each side. The picture’s width is 5 less than twice the picture’s height. The entire area of the frame with picture included is 221 square inches. What are the dimensions of the picture?