## 1.3 - Transformations in the Coordinate Plane

### Coordinate Transformations

#### Transformation Types:

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#### Translations

1. Plot the points and create the pre-image triangle with vertices M(–2, 3), N(–3, 5), and P(–5, 2). Then, determine the coordinates M′, N′, and P′ of the image triangle that has been translated right 6 and down 3. Explain what you did to each of the coordinates.

   \[
   (-2 + 6, 3 - 3) \rightarrow M'(4, 0) \\
   (-3 + 6, 5 - 3) \rightarrow N'(3, 2) \\
   (-5 + 6, 2 - 3) \rightarrow P'(1, -1)
   \]

#### Reflections

2. List the coordinates of the triangle ABC

   \[
   A(1, 2) \\
   B(4, 1) \\
   C(5, 4)
   \]

3. Reflect the triangle ABC over the y-axis and list the coordinates of the vertex A′, B′, and C′. Describe what happened to the coordinates from the pre-image to the image.

   \[
   A'(1, 2) \\
   B'(-4, 1) \\
   C'(-5, 4)
   \]

4. Reflect the triangle ABC over the x-axis and list the coordinates of the vertex A′, B′, and C′. Describe what happened to the coordinates from the pre-image to the image.

   \[
   A'(1, -2) \\
   B'(4, -1) \\
   C'(-5, -4)
   \]
Reflections
5. List the coordinates of the quadrilateral ABCD

\[ A(-2, 3) \]
\[ B(1, 2) \]
\[ C(1, 3) \]
\[ D(0, 3) \]

6. Reflect the quadrilateral ABCD over the line \( x = 3 \) and list the coordinates of the vertex
A’, B’, C’, and D’.

\[ A'(5, 3) \]
\[ B'(5, 2) \]
\[ C'(5, 5) \]
\[ D'(6, 3) \]

Reflections
7. List the coordinates of the quadrilateral KLMN

\[ K(1, 2) \]
\[ L(-2, 3) \]
\[ M(0, 3) \]
\[ N(2, 5) \]

8. Reflect the quadrilateral KLMN over the line \( y = x \) and list the coordinates of the vertex
K’, L’, M’, and N’. Describe what happened to the coordinates from the pre-image to the image.

\[ K'(2, 1) \]
\[ L'(3, -2) \]
\[ M'(3, 0) \]
\[ N'(5, 2) \]

Reflections
9. If the point A is located at \((-3, 2)\) and A’ is the image of A after being reflected over the \( x \)-axis, what are the coordinates of A’?

A’ \((-3, -2)\)

10. If the point B is located at \((-4, -1)\) and B’ is the image of B after being reflected over the \( y \)-axis, what are the coordinates of B’?

B’ \((4, -1)\)

11. If the point C is located at \((2, -3)\) and C’ is the image of C after being reflected over the line \( y = x \), what are the coordinates of C’?

C’ \((-3, 2)\)
Rotations

12. List the coordinates of the quadrilateral ABCD.

   \[ A(2,1) \quad C(5,3) \]
   \[ B(2,3) \quad D(1,4) \]

13. Rotate the quadrilateral \textbf{ABCD about the origin by 90°} and list the coordinates of the vertex A’, B’, C’, and D’. Describe what happened to the coordinates from the pre-image to the image.

   \[ A'(-1,2) \quad C'(-3,5) \]
   \[ B(-3,2) \quad D'(-4,1) \]

14. Rotate the quadrilateral \textbf{ABCD about the origin by 180°} and list the coordinates of the vertex A’, B’, C’, and D’. Describe what happened to the coordinates from the pre-image to the image.

   \[ A'(-2,-1) \quad C'(-5,-3) \]
   \[ B(-2,-3) \quad D'(-1,-4) \]

15. Rotate the quadrilateral \textbf{ABCD about the origin by 270°} and list the coordinates of the vertex A’, B’, C’, and D’. Describe what happened to the coordinates from the pre-image to the image.

   \[ A'(1,-2) \quad C'(3,-5) \]
   \[ B'(5,-2) \quad D'(4,-1) \]

16. If the point A is located at \((-3,2)\) and A’ is the image of A after being rotated about the origin by 270°. What are the coordinates of A’?

\[ (2,1) \]
Dilations

17. List the coordinates of the quadrilateral ABCD.
   \[
   \begin{align*}
   &A(1, 1) \quad C(3, 2) \\
   &B(1, 3) \quad D(2, 2)
   \end{align*}
   \]

18. Dilate the quadrilateral ABCD by a scale factor of 3 from the origin and list the coordinates of the vertex A', B', C', and D'. Describe what happened to the coordinates from the pre-image to the image.
   \[
   \begin{align*}
   &A'(3, 3) \quad C'(9, 6) \\
   &B'(3, 9) \quad D'(6, 16)
   \end{align*}
   \]

19. List the coordinates of the quadrilateral ABCD.
   \[
   \begin{align*}
   &A(-2, -2) \quad C(4, 3) \\
   &B(2, -4) \quad D(-4, -1)
   \end{align*}
   \]

20. Dilate the quadrilateral ABCD by a scale factor of ½ from the origin and list the coordinates of the vertex A', B', C', and D'. Describe what happened to the coordinates from the pre-image to the image.
   \[
   \begin{align*}
   &A'(-1, -1) \quad C'(2, 1.5) \\
   &B'(1, -2) \quad D'(-2, 2)
   \end{align*}
   \]

21. If the point A is located at (3, -2) and A' is the image of A after being dilated by a scale factor of 5 from the origin. What are the coordinates of A'?
22. Dilate the triangle ABC by a scale factor of 4 from the point D and list the coordinates of the vertex A’, B’, and C’.

Create a list of all of the basic coordinate transformation rules:

**Translation:** \((x, y) \rightarrow (x+h, y+k)\) / TRANSLATE HORIZONTALLY BY \(h\) VERTICALLY BY \(k\)

\[\text{e.g. Translate } (1, 2) \text{ left 3 and up 4 } \rightarrow (1-3, 2+4) = (-2, 6)\]

**Reflect Over Y-Axis**
\((x, y) \rightarrow (-x, y)\)

**Reflect Over X-Axis**
\((x, y) \rightarrow (x, -y)\)

**Reflect Over Line \(y=x\)**
\((x, y) \rightarrow (y, x)\)

**Rotate 90° About Origin**
\((x, y) \rightarrow (-y, x)\)

**Rotate 180° About Origin**
\((x, y) \rightarrow (-x, -y)\)

**Rotate 270° About Origin**
\((x, y) \rightarrow (y, -x)\)

**Dilation of a Scale Factor of \(M\) from Origin.**
\((x, y) \rightarrow (M \cdot x, M \cdot y)\)
Translations

23. Point A (-3, 3) is on \( \overline{AB} \). A translation moves the point A to its image \( A'(1,1) \).

\[
\begin{align*}
A & : (x_1, y_1) = (-3, 3) \\
A' & : (x_2, y_2) = (1, 1)
\end{align*}
\]

All points will be translated by the same distance.

What is the distance, in units, between any point on \( \overline{AB} \) and its image?

We could use the distance formula to find the distance between point A and point A'.

\[
d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}
\]

\[
d = \sqrt{(1 - (-3))^2 + (1 - 3)^2} = \sqrt{4^2 + (-2)^2}
\]

\[
d = \sqrt{16 + 4} = \sqrt{20} = \sqrt{4} \cdot \sqrt{5} = 2\sqrt{5}
\]

\[
2.25
\]

\[
2.25
\]

\[
\approx 4.47
\]

Alternatively, we could just use the Pythagorean theorem to find the distance between point A and point A'.

\[
2^2 + 4^2 = c^2
\]

\[
4 + 16 = c^2
\]

\[
\sqrt{20} = c
\]

\[
\sqrt{20} = c
\]

\[
2\sqrt{5} = c
\]

\[
4.47 \approx c
\]