### Name all of the properties of a parallelogram and its diagonals.

1. Opposite Sides are parallel
2. Opposite Sides are congruent
3. Opposite Angles are congruent
4. Consecutive Angles are supplementary
5. Diagonals bisect each other

### The properties of a rectangle and its diagonals:

1. All angles are right
2. Opposite Sides are parallel
3. Opposite Sides are congruent
4. Diagonals bisect each other
5. Diagonals are congruent

### The properties of a rhombus and its diagonals:

1. Opposite angles are congruent
2. Opposite Sides are parallel
3. All Sides are congruent
4. Consecutive angles are supplementary
5. Diagonals are perpendicular
6. Diagonals bisect interior angles.

### The properties of a square and its diagonals:

1. All angles are right
2. Opposite Sides are parallel
3. All Sides are congruent
4. Diagonals bisect each other
5. Diagonals are perpendicular
6. Diagonals bisect angles
7. Diagonals are congruent
Find the value of \( x \) in each diagram below using properties of quadrilaterals.

1. \( (8x + 5)^\circ + (3x + 10)^\circ = 180^\circ \)
   \[ 11x + 15 = 180 \]
   \[ 11x = 165 \]
   \[ x = 15 \]

2. \( 8x - 2 = 6x + 12 \)
   \[ 2x = 14 \]
   \[ x = 7 \]

3. \( a + 90 + 50 = 180 \)
   \[ a + 140 = 180 \]
   \[ a = 40 \]

4. \( AC = 9x + 4 \)
   \( BD = 6x + 16 \)
   \[ 9x + 4 = 6x + 16 \]
   \[ 3x = 12 \]
   \[ x = 4 \]

5. \( a^2 + b^2 = c^2 \)
   \[ 8^2 + 8^2 = c^2 \]
   \[ 2 \cdot 8^2 = 64 \]
   \[ x = \sqrt{64} = 8 \]

6. \( (2x)^2 + (x)^2 = 45^2 \)
   \[ 4x^2 + x^2 = 2025 \]
   \[ 5x^2 = 2025 \]
   \[ x^2 = 405 \]
   \[ x = \sqrt{405} = 9\sqrt{5} \]
   \[ x \approx 20.12 \]
Plot points \( A(-3, -1), B(-1, 2), C(4, 2), \) and \( D(2, -1) \).

1. What specialized geometric figure is quadrilateral \( ABCD \)? Support your answer mathematically.

   \[ \text{ABCD IS A PARALLELGRAM} \]
   - \( \text{SLOPE AB} = \frac{-1-2}{-3+1} = -\frac{3}{2} = \text{SLOPE DC} \)
   - \( \text{SLOPE AD} = \frac{0-(-1)}{0-2} = \frac{1}{2} = \text{SLOPE BC} \)

2. Draw the diagonals of \( ABCD \). Find the coordinates of the midpoint of each diagonal. What do you notice?

   \[
   \text{MIDPOINT BD} \quad \text{MIDPOINT AC} \\
   \left( \frac{-1+2}{2}, \frac{-1+1}{2} \right) = \left( 0, \frac{0}{2} \right) = \left( 0, \frac{0}{2} \right)
   \]

3. Find the slopes of the diagonals of \( ABCD \). What do you notice?

   \[
   \text{SLOPE BD} \quad \text{SLOPE AC} \\
   \frac{(y_2-y_1)}{(x_2-x_1)} = \frac{(-1)-2}{(-1)-1} = \frac{-3}{-2} = \frac{3}{2} \quad \frac{(y_2-y_1)}{(x_2-x_1)} = \frac{(2)-(-1)}{(4)-(-1)} = \frac{3}{5}
   \]

4. The diagonals of \( ABCD \) create four small triangles. Are any of these triangles congruent to any of the others? Why or why not?

   \( \triangle AMD \cong \triangle CMB \) BY SAS
   \( \triangle AMB \cong \triangle CMD \) BY SAS

Plot points \( E(1, 2), F(2, 5), G(4, 3) \) and \( H(5, 6) \).

5. What specialized geometric figure is quadrilateral \( EFHG \)? Support your answer mathematically using two different methods.

   \[
   \text{EFHG IS A RHOMBUS} \\
   FE = FH = HG = EG = \sqrt{1^2+3^2} = \sqrt{10}
   \]

6. Draw the diagonals of \( EFHG \). Find the coordinates of the midpoint of each diagonal. What do you notice?

   \[
   \text{MIDPOINT EG} \quad \text{MIDPOINT FH} \\
   \left( \frac{2+4}{2}, \frac{5+3}{2} \right) = \left( 3, 4 \right) \quad \left( \frac{1+5}{2}, \frac{2+6}{2} \right) = \left( 3, 4 \right)
   \]

7. Find the slopes of the diagonals of \( EFHG \). What do you notice?

   \[
   \text{SLOPE EG} \quad \text{SLOPE FH} \\
   \frac{(y_2-y_1)}{(x_2-x_1)} = \frac{(3)-2}{(4)-2} = \frac{1}{2} \quad \frac{(y_2-y_1)}{(x_2-x_1)} = \frac{(5)-2}{(5)-1} = \frac{3}{4}
   \]

8. The diagonals of \( EFHG \) create four small triangles. Are any of these triangles congruent to any of the others? Why or why not?

   \( \triangle FME \cong \triangle FMH \cong \triangle GMH \cong \triangle GME \) BY SSS.
Plot points P(4, 1), W(-2, 3), M(2,-5), and K(-6, -4).

9. What specialized geometric figure is quadrilateral PWKM? Support your answer mathematically.

\[ PW = \sqrt{4^2 + 2^2} = \sqrt{20} \approx 4.47 \]
\[ KW = \sqrt{2^2 + 5^2} = \sqrt{29} \approx 5.39 \]
\[ WM = \sqrt{4^2 + 8^2} = \sqrt{80} \approx 8.94 \]

10. Draw the diagonals of PWKM. Find the coordinates of the midpoint of each diagonal. What do you notice?

\[ KP = \left( \frac{4+6}{2}, \frac{1+1}{2} \right) = (5, 1) \]
\[ WM = \left( \frac{2+6}{2}, \frac{-5+4}{2} \right) = (4, -0.5) \]

11. Find the lengths of the diagonals of PWKM. What do you notice?

\[ KP = \sqrt{(5-4)^2 + (1-1)^2} = \sqrt{1} = 1 \]
\[ WM = \sqrt{(4-2)^2 + (-0.5+5)^2} = \sqrt{30.25} \approx 5.51 \]

12. Find the slopes of the diagonals of PWKM. What do you notice?

\[ \text{Slope } KP = \frac{1-1}{5-4} = 0 \]
\[ \text{Slope } WM = \frac{-0.5-5}{4-2} = -\frac{5.5}{2} = -2.75 \]

13. The diagonals of ABCD create four small triangles. Are any of these triangles congruent to any of the others? Why or why not?

\[ \triangle KOM \cong \triangle KOW \]
\[ \triangle PON \cong \triangle POM \]

Plot points A(1, 0), B(-1, 2), and C(2, 5).

14. Find the coordinates of a fourth point D that would make ABCD a rectangle. Justify that ABCD is a rectangle.

\[ \text{Slope } AD = \frac{y_2-y_1}{x_2-x_1} = \frac{2-0}{1-1} = \text{undefined} = \pm \infty \]
\[ \text{Slope } AB = \frac{y_2-y_1}{x_2-x_1} = \frac{0-0}{1+1} = 0 = \pm 0 \]
\[ \text{Slope } BC = \frac{y_2-y_1}{x_2-x_1} = \frac{5-2}{2+1} = \frac{3}{3} = 1 \]
\[ \text{Slope } CD = \frac{y_2-y_1}{x_2-x_1} = \frac{-2-5}{2+1} = \frac{-7}{3} = \pm \frac{7}{3} \]

15. Find the coordinates of a fourth point D that would make ABCD a parallelogram that is not also a rectangle. Justify that ABCD is a parallelogram but is not a rectangle.

\[ \text{Slope } AB = \frac{y_2-y_1}{x_2-x_1} = \frac{2-0}{1+1} = \frac{1}{2} = \pm 0.5 \]
\[ \text{Slope } AC = \frac{y_2-y_1}{x_2-x_1} = \frac{5-0}{2+1} = \frac{5}{3} \]
\[ \text{Slope } CD = \frac{y_2-y_1}{x_2-x_1} = \frac{-2-5}{2+1} = \frac{-7}{3} = \pm \frac{7}{3} \]
\[ \text{Slope } BD = \frac{y_2-y_1}{x_2-x_1} = \frac{7-2}{0+1} = \frac{5}{1} = 5 \]