### Defining a triangle by its SIDES:

**SCALENE Triangle**
- A triangle with 3 sides all of different lengths is referred to as a **Scalene Triangle**.

**ISOSCELES Triangle**
- A triangle with exactly 2 congruent sides is referred to as an **Isosceles Triangle**.

**EQUILATERAL Triangle**
- A triangle with all sides being congruent is referred to as an **Equilateral Triangle**.

(Equilateral triangles are also always **equiangular**.

### Defining a triangle by its ANGLES:

**ACUTE Triangle**
- A triangle with all 3 interior angles that are acute (i.e. angle measure less than 90°) is referred to as an **Acute Triangle**.

**RIGHT Triangle**
- A triangle that has exactly 1 right interior angle (i.e. an angle that measures exactly 90°) is referred to as a **Right Triangle**.

**OBTUSE Triangle**
- A triangle that has exactly 1 obtuse interior angle (i.e. angle measure greater than 90°) is referred to as an **Obtuse Triangle**.

### Classify each triangle below by its sides and by its angles.

1. ![Scalene Triangle](image1)
   - Classify by SIDE: **ISOSCELES Δ**
   - Classify by ANGLE: **ACUTE Δ**

2. ![Isosceles Triangle](image2)
   - Classify by SIDE: **SCALENE Δ**
   - Classify by ANGLE: **RIGHT Δ**

3. ![Equilateral Triangle](image3)
   - Classify by SIDE: **EQUILATERAL Δ**
   - Classify by ANGLE: **ACUTE or EQUILANGULAR**

4. Briefly explain why it is impossible to have an obtuse equilateral triangle.

Both of these cannot be true at the same time for a single triangle. One of the angles is greater than 90°. All of the angles must measure 60°.
**Triangle Inequality Theorem:**

The sum of the lengths of any two sides of a triangle is greater than the length of the third side.

This statement might be easier to understand in context.

Michael drives to school from his house. He travels 5 miles on the trip. His friend Janet drives from their school to a department store. She travels 3 miles on her trip.

At most, how far is the same department store away from Michael’s house?

\[ 5 + 3 = 8 \text{ miles} \]

What is the closest the department store could be to Michael’s house (assume the paths are linear)?

\[ 5 - 3 = 2 \text{ miles} \]

5. Two friends start from the same place at a park to go for a jog. They take different paths both paths are nearly straight. Jennifer takes the beginner’s path and Zach takes the intermediate path. An hour and half later, Jenny is 4 miles away from the point where they started and Zach is 7 miles from where they started.

- a. If nothing is known about the path’s direction, what is the furthest apart the two friends could be based on the triangle inequality theorem?

\[ 7 + 4 = 11 \text{ miles} \]

- b. If nothing is known about the path’s direction, what is the nearest the two friends could be based on the triangle inequality theorem?

\[ 7 - 4 = 3 \text{ miles} \]

6. Determine the range of possible lengths of the last unknown side of a triangle given the lengths of two sides of a triangle. Write an inequality statement using the variable.

- a. Triangle ABC

- b. Triangle GHI has sides:
  - GH = 2 feet
  - HI = 4 feet
  - GI = y feet

\[ 4 - 2 = 2 \text{ ft} \]
\[ 4 + 2 = 6 \text{ ft} \]

\[ 2 < y < 6 \]

- c. Triangle RST

\[ 47 - 38 = 9 \text{ mm} \]
\[ 47 + 38 = 85 \text{ mm} \]

\[ 9 < r < 85 \]

7. Which of the following sets of measures could be the 3 lengths of the sides of a triangle? If they cannot represent the sides of a triangle explain why.

- a. 5 cm, 9 cm, 3 cm

Not a triangle

\[ 5 + 9 = 14 \text{ cm} \]
\[ 9 + 3 = 12 \text{ cm} \]
\[ 5 + 3 = 8 \times 9 \]

5 + 3 is smaller than the third side

- b. 8 m, 4m, 10m

These could be a triangle

\[ 8 + 4 = 12 > 10 \checkmark \]
\[ 8 + 10 = 18 > 4 \checkmark \]
\[ 4 + 10 = 14 > 8 \checkmark \]

- c. 6 ft., 9 ft., 3 ft.

Not a triangle

\[ 6 + 9 = 15 > 3 \checkmark \]
\[ 9 + 3 = 12 > 6 \checkmark \]
\[ 6 + 3 = 9 = 9 \times \]

6 + 3 is exactly equal to the third side
Angle-Side Triangle Inequality Theorem: If one side of a triangle is longer than another side, then the angle opposite the longer side will have a greater degree measure than the angle opposite the shorter side. The converse is true as well. (i.e. the shortest side of a triangle is always opposite the shortest angle of the triangle or the largest side is opposite the largest angle.)

8. Determine which side of each diagram is the longest and which is the shortest based on the angle measures.

<table>
<thead>
<tr>
<th>Diagram</th>
<th>Longest Side</th>
<th>Shortest Side</th>
</tr>
</thead>
<tbody>
<tr>
<td>a.</td>
<td>AB</td>
<td>BC</td>
</tr>
<tr>
<td>b.</td>
<td>HJ</td>
<td>HK = JK</td>
</tr>
<tr>
<td>c.</td>
<td>FE</td>
<td>DG</td>
</tr>
</tbody>
</table>

9. Determine which interior angle of each diagram is the largest and which is the smallest based on the side measures.

<table>
<thead>
<tr>
<th>Diagram</th>
<th>Largest Angle</th>
<th>Smallest Angle</th>
</tr>
</thead>
<tbody>
<tr>
<td>a.</td>
<td>( \angle B )</td>
<td>( \angle C )</td>
</tr>
<tr>
<td>b.</td>
<td>( \angle Z )</td>
<td>( \angle X )</td>
</tr>
<tr>
<td>c.</td>
<td>( \angle KLM \neq \angle NMO )</td>
<td>( \angle MNO )</td>
</tr>
</tbody>
</table>

10. Write an inequality statement for all three sides.

11. Write an inequality statement for all three angles.

12. Using this more advanced inequality theorem, determine the most restrictive range of values for the unknown side \( g \).

\[ 30 \text{ mm} \leq g \leq 38 \text{ mm} \]
\[ S_1 + L + 58 = 180 \]
\[ S_2 + L + 62 = 180 \]

\[ S_1 + 58 = S_2 + 62 \]

\[ S_1 = S_2 + 4 \]