5.5 - Connecting Algebra & Geometry using Coordinates
Applications and Proofs with Coordinates

Name:

Areas:
1. Find the area of the rectangles shown in each graph below.

A. \[ A = \text{Area} = (\text{B})(\text{H}) = 8 \times 4 = 32 \text{ sq units} \]

B. \[ BC = \sqrt{(5-3)^2 + (-1-3)^2} = \sqrt{16 + 16} = \sqrt{32} \]
\[ AB = \sqrt{(-3-4)^2 + (-3-1)^2} = \sqrt{25 + 16} = \sqrt{41} \]
\[ A = \text{Area} = (\text{B})(\text{H}) = \sqrt{32} \times \sqrt{41} = 34 \text{ sq units} \]

2. Find the area of the triangles shown in each graph below.

A. \[ \text{BASE} (AC) = 3 \text{ units} \]
\[ \text{HEIGHT} = 2 \text{ units} \]
\[ A \triangle = \frac{1}{2} \cdot \text{b} \cdot \text{h} = \frac{3 \cdot 2}{2} = 3 \text{ sq units} \]

B. \[ \text{BASE} (AB) = \sqrt{(3-1)^2 + (-1-2)^2} = \sqrt{4 + 9} = \sqrt{13} \]
\[ \text{HEIGHT} (AC) = \sqrt{(5-1)^2 + (-2-2)^2} = \sqrt{36 + 16} = \sqrt{52} \]
\[ A \triangle = \frac{1}{2} \cdot \text{b} \cdot \text{h} = \frac{\sqrt{13} \cdot \sqrt{52}}{2} = 13 \text{ sq units} \]
3. Find the area of the circles shown in each graph below. (AB is a diameter represented in both circles.)

A. \[ A \bigcirc = \pi r^2 \]

\[
\text{Midpoint (AB): } \left( \frac{-5+3}{2}, \frac{-1+1}{2} \right) = (-1, -1)
\]

\[
A = \pi (4)^2 = 16\pi \text{ sq units}
\approx 50.27 \text{ sq units}
\]

B. \[ A \bigcirc = \pi \left( \frac{1+2}{2} \right)^2 = 13\pi \text{ sq units}
\approx 40.84 \text{ sq units}
\]

**Perimeters:**

4. Find the perimeter of the rectangles shown in each graph below.

A. \[ P = 8 + 4 + 8 + 4 \]

\[ \text{Perimeter} = 24 \text{ units} \]

B. \[ \sqrt{68} + \sqrt{17} + \sqrt{17} + \sqrt{68} = 2\sqrt{17} + 2\sqrt{68} \]

\[ \text{Perimeter} = 2\sqrt{17} + 4\sqrt{17} = 6\sqrt{17} \text{ units} \]
5. Prove the triangle ABC shown in the graph is a RIGHT triangle using the coordinates of its vertices: A(−5, 1), B(5,1), and C(4,4).

\[
\text{Slope (AC)} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{4 - 1}{5 - (-5)} = \frac{3}{10} = \frac{3}{3} \\
\text{Slopes suggest that} \quad \overline{AC} \perp \overline{BC} \quad \text{that means} \quad \triangle ABC \text{ is a RIGHT TRIANGLE}
\]

\[
\text{Slope (BC)} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{4 - 1}{5 - 4} = -3
\]

6. Prove the triangle ABC shown in the graph is an ISOSCELES triangle using the coordinates of its vertices: A(−3, 0), B(1, −4), and C(5,4).

\[
\text{Length (AC)} = \sqrt{(5-(-3))^2 + (4-0)^2} = \sqrt{64 + 16} = \sqrt{80} \quad \uparrow
\]

\[
\text{Length (AB)} = \sqrt{(1-(-3))^2 + (-4-0)^2} = \sqrt{16 + 16} = \sqrt{32}
\]

\[
\text{Length (BC)} = \sqrt{(1-5)^2 + (-4-4)^2} = \sqrt{16 + 64} = \sqrt{80} \\
\]

\[
\text{Only 2 sides are congruent, so} \ 	riangle ABC \text{ is an ISOSCELES TRIANGLE}
\]

7. Prove the quadrilateral ABCD shown in the graph is a PARALLELOGRAM using the coordinates of its vertices: A(−4, 3), B(2, 1), C(4, −4), and D(−2, −2).

\[
\text{Slope (AB)} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{1 - 3}{2 - (-4)} = \frac{-2}{6} = -\frac{1}{3} \quad \text{Sides are parallel}
\]

\[
\text{Slope (CD)} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{2 - (-4)}{-2 - 4} = \frac{6}{-6} = -\frac{1}{3} \quad \text{Sides are parallel}
\]

\[
\text{Slope (AD)} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-2 - 3}{-2 - (-4)} = \frac{-5}{2}
\]

\[
\text{Slope (BC)} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-4 - 1}{4 - 2} = \frac{-5}{2}
\]
8. Prove the rectangle ABCD shown in the graph has **congruent** diagonals using the coordinates of its vertices: A(–4, 1), B(–3, –3), C(5, –1), and D(4, 3).

   \[
   \text{LENGTH (AC)} = \sqrt{(5-(-4))^2 + (-1-1)^2} = \sqrt{81 + 4} = \sqrt{85}
   \]

   \[
   \text{LENGTH (BD)} = \sqrt{(4-(-3))^2 + (3-(-3))^2} = \sqrt{49 + 36} = \sqrt{85}
   \]

9. Prove the parallelogram ABCD shown in the graph has diagonals that bisect each other using the coordinates of its vertices: A(–2, 4), B(–4, –1), C(2, 0), and D(4, 3).

   \[
   \frac{\sqrt{(0-(-4))^2 + (2-1)^2}}{\sqrt{16 + 1}} = \frac{\sqrt{(4-0)^2 + (3-2)^2}}{\sqrt{16 + 1}} = \frac{\sqrt{17}}{\sqrt{17}} = 1
   \]

10. Prove the quadrilateral ABCD shown in the graph is a **RECTANGLE** using the coordinates of its vertices: A(–4, 3), B(2, 1), C(4, –4), and D(–2, –2) and showing that consecutive sides are perpendicular.

   \[
   \text{SLOPE (AB)} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{3 - 2}{-1 - 4} = \frac{1}{3} = -1
   \]

   \[
   \text{SLOPE (AD)} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{0 - 3}{4 - 1} = \frac{-3}{3} = -3
   \]

   \[
   \text{SLOPE (BC)} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{1 - 4}{1 - 2} = \frac{-3}{3} = 1
   \]

   \[
   \text{SLOPE (CD)} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-4 - 1}{-2 - (-4)} = \frac{-5}{2} = -1
   \]

   **Since any 2 consecutive sides have negative reciprocal slopes, the consecutive sides are perpendicular. So, all interior angles are right angles.**
2. The coordinates of Quadrilateral $QRST$ are $Q(-3, 1)$, $R(-2, 4)$, $S(4, 2)$, $T(3, -1)$
   a. Algebraically verify that the Quadrilateral is a Rectangle by showing that consecutive sides are perpendicular.

   \[ M_{QR} : \frac{4 - 1}{-2 - (-3)} = \frac{3}{1} \]
   \[ M_{ST} : \frac{-1 - 2}{3 - (-4)} = \frac{-3}{1} = 3 \]
   \[ \text{PERPENDICULAR} \]

   \[ M_{RS} : \frac{2 - 4}{4 - (-2)} = \frac{-2}{6} = -\frac{1}{3} \]
   \[ M_{TQ} : \frac{-1 - 1}{3 - (-3)} = \frac{-2}{6} = -\frac{1}{3} \]
   \[ \text{PERPENDICULAR} \]

   b. Algebraically verify the diagonals $QS$ and $RT$ are congruent.

   \[ \text{LENGTH OF QS} = \sqrt{(-4 - 3)^2 + (2 - 1)^2} = \sqrt{49 + 1} = \sqrt{50} = 5\sqrt{2} \]

   \[ \text{LENGTH OF RT} = \sqrt{(-1 - 4)^2 + (3 - (-2))^2} = \sqrt{25 + 25} = \sqrt{50} = 5\sqrt{2} \]

   \[ \text{LENGTHS ARE THE SAME} \]

3. Given that the 3 points shown at the right are

   vertices of a parallelogram, find all of the

   possible points of the fourth point that would

   create a parallelogram. There are 3 of them

   draw each one.