Algebra 2

Unit 2: Operations with Polynomials

MAtTHew M. Winking

(5x - 3)(4x + 1)

20x^2 + 5x - 12x - 3

20x^2 - 7x - 3
## Sec 2.1 – Operations with Polynomials
### Polynomial Classification and Operations

<table>
<thead>
<tr>
<th>Name</th>
<th>Examples</th>
<th>Non-Examples</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Monomial</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(one term)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1. $3x^4$</td>
<td>degree: 4 or quartic</td>
<td>1. $2x^{-4}$</td>
</tr>
<tr>
<td>2. $a^2$</td>
<td>degree: 2 or quadratic</td>
<td>2. $5\sqrt{m}$</td>
</tr>
<tr>
<td>3. 5</td>
<td>degree: 0 or constant</td>
<td>3. $3t^2$</td>
</tr>
<tr>
<td><strong>Binomial</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(two terms)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1. $2n^3 - n$</td>
<td>degree: 3 or cubic</td>
<td>1. $\frac{2x+1}{x}$</td>
</tr>
<tr>
<td>2. $p - 3$</td>
<td>degree: 1 or linear (monic)</td>
<td>2. $\sqrt{c^3 - 2}$</td>
</tr>
<tr>
<td>3. $-3a^3b^4 + a^4b^5$</td>
<td>degree: 9 or nonic</td>
<td></td>
</tr>
<tr>
<td><strong>Trinomial</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(three terms)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1. $-2x^3 + 2x - 3$</td>
<td>degree: 3 or cubic</td>
<td>1. $x^{-3} + 2x - 5$</td>
</tr>
<tr>
<td>2. $d(d^2 + 2d^4 - 2)$</td>
<td>degree: 5 or quintic</td>
<td>2. $2x^2 + 3x - 5$</td>
</tr>
<tr>
<td><strong>Polynomial</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(one or more terms)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1. $3x^4 + 2x^3 - 5x + 1$</td>
<td>degree: 4 or quartic</td>
<td>1. $3q^3 + \frac{p}{q}$</td>
</tr>
<tr>
<td>2. $\frac{1}{2}x^2 + \sqrt{3} x^3 - 6x^4 + 1x - 3$</td>
<td>degree: 4 or quartic</td>
<td>2. $2x^3 + 3\sqrt{x}$</td>
</tr>
</tbody>
</table>

1. **EXPAND and SIMPLIFY**
   a. $(7x + 3) - (2 - 2x)$
   b. $(5x^3 - 3x^4 - 2x - 9x^2 - 2) + (3x^3 + 2x^2 - 5x - 7)$
   c. $3(x + 5) + 8x$                             
   d. $-2(3x + 2y) - (5x - 6y) + 2x - 7$          
   e. $(2x^2 + 5x) - (6x^2 - 2x)$                 
   f. $(2x^3 + 5x - 8) + (5x^3 - 9x^2 - 11x + 5)$  
   g. $(x + 3)(x + 5)$                           
   h. $(2x - 5)^2$
(1 Continued). EXPAND and SIMPLIFY

i. $4y^2(y^2 + 2y)$

j. $-6y^2(3y^2 - 2y - 7)$

k. $(2x + 3)(3x - 5)$

l. $(2a + 3)(\alpha^2 + 2\alpha - 4)$

m. $(x - 2)(x^2 + 4)(2x + 3)$

n. \[\begin{align*}
\text{Area} &= 3x + 8 \\
\text{Perimeter} &= 5x + 2
\end{align*}\]

Determine an expression that represents:

Perimeter =

Area =

Determine an expression that represents:

Perimeter =

Area =

M. Winking  Unit 2-1 page 28
1. Expand each of the following.

a. \((a + b)^0\)

b. \((a + b)^1\)

c. \((a + b)^2\)

d. \((a + b)^3\)

e. \((a + b)^4\)

f. \((a + b)^5\)

2. Create Pascal’s triangle to the 7th row.
3. Using Pascal’s Triangle expand \((2a - 3b)^4\)

The Binomial Theorem permits you to determine any row of Pascal’s Triangle Explicitly.

The **Binomial Theorem** is shown below:

\[
(a + b)^n = \binom{n}{0}(a)^n(b)^0 + \binom{n}{1}(a)^{n-1}(b)^1 + \binom{n}{2}(a)^{n-2}(b)^2 + \ldots \ldots \ldots + \binom{n}{n-1}(a)^1(b)^{n-1} + \binom{n}{n}(a)^0(b)^n
\]

**EXAMPLE:** Expand \((3x - 2y)^5\)

\[
\begin{align*}
\binom{5}{0}(3x)^5(-2y)^0 & = 243x^5 \\
\binom{5}{1}(3x)^4(-2y)^1 & = \frac{243}{5}x^4y \\
\binom{5}{2}(3x)^3(-2y)^2 & = \frac{243}{25}x^3y^2 \\
\binom{5}{3}(3x)^2(-2y)^3 & = \frac{243}{125}x^2y^3 \\
\binom{5}{4}(3x)^1(-2y)^4 & = \frac{243}{625}xy^4 \\
\binom{5}{5}(3x)^0(-2y)^5 & = 32y^5
\end{align*}
\]

4. Using the Binomial Theorem expand \((3x - 2y)^6\)
Use the Binomial Theorem to answer the following:

5. What is just the 4th term of \((3c + 4d)^7\)

6. What is just the 7th term of \((3q - 2p)^6\)

7. What is just the coefficient of the 3rd term of \((3t + 5m)^8\)

8. Which term of \((5a - 3b)^9\) could be represented by \(\binom{9}{7}(5a)^7(-3b)^2\)

9. The Binomial Theorem also has some applications in counting. For example if you wanted to know the probability of 6 coins being flipped and the probability that 5 of the flipped coins will land on heads by expanding.
   First, expand \((h + t)^6\) using which ever method you would prefer.
   Each coefficient represents the number of different ways you can flip a the 6 coins that way.
   (e.g. 15h^4t^2 suggests there are 15 different ways the 6 coins could land with 4 heads up and 2 tails up)
   a. Determine the probability of having 5 coins land heads up.
   b. Determine the probability of having 2 or more tails landing heads up.
1. Divide each of the following polynomials by the suggested monomial.

   a. \( \frac{32a^3 + 24a}{8a^3} \)

   b. \( \frac{36x^5 + 72x^3 + 48x^2}{6x^2} \)

   c. \( \frac{12m^5 + 20m^4 + 32m^2}{4m^3} \)

2. (REVIEW) Complete the following long division problem: \( 385274 \div 12 \)

   \[
   \begin{array}{c|cccc}
   \hline
   12 & 3 & 8 & 5 & 2 & 7 & 4 \\
   \hline
   \end{array}
   \]

3. Use long division to divide the following polynomials. \( \left( x^4 + 5x^3 + 3x^2 - 8x + 3 \right) \div (x + 3) \)

   \[
   \begin{array}{c|ccccc}
   \hline
   x + 3 & x^4 & + & 5x^3 & + & 3x^2 & - & 8x & + & 3 \\
   \hline
   \end{array}
   \]

4. Use long division to divide \( (2x^4 - 7x^3 + 10x^2 - 6x - 3) \) by \( (x - 2) \)

5. Use long division to divide: \( \frac{2x^4 - 3x^3 - x^2 + 8x - 6}{2x + 3} \)
6. Use long division to divide \( (4x^4 + 3x^2 + 2x - 2) \) by \((2x - 1)\)

7. Use long division to divide:
\[
\frac{x^5 + 5x^4 - x^3 - 22x^2 + x + 6}{x^2 + 3x - 2}
\]

8. Use long division to find the quotient of \(3x^3 - 12\) and \(x^2 + 2x + 3\)

9. Use long division to determine.
\[
(6x^4 + 8x^3 - 11x^2 - 7x + 6) \div (2x^2 + 2x - 3)
\]

10. Rewrite \(2x^3 + 3x^2 + 4x - 5\) as a nested polynomial.

11. Use the nested polynomial to easily evaluate \(2x^3 + 3x^2 + 4x - 5\) when \(x = 2\).

12. Use the following format to quickly evaluate \(2x^3 + 3x^2 + 4x - 5\) when \(x = 2\).

\[
\begin{array}{cccc}
2 & \downarrow & 2 & 3 & 4 & -5 \\
& & 2 \\
\end{array}
\]
13. Use long division to find the following quotient: \( \left( 2x^3 + 3x^2 + 4x - 5 \right) ÷ (x - 2) \)

(Compare the answer from problem #12 with problem #11.)

14. Use synthetic division to divide
\[ \left( x^4 + 7x^3 + 17x^2 + 13x - 6 \right) ÷ (x + 3) \]

15. Use synthetic division to divide
\[ \left( x^4 + 3x^3 + 2x^2 + x - 6 \right) ÷ (x + 2) \]

16. Use synthetic division to divide
\[ \left( 2x^4 - 5x^3 - 7x - 6 \right) ÷ (x - 3) \]

17. Use synthetic division to evaluate
\[ 2x^3 + 3x^2 - 4x - 2 \] at \( x = 3 \)

18. Use synthetic division find the remainder of
\[ \left( 3x^4 + 10x^3 - 6x^2 + 5x - 7 \right) ÷ (x + 4) \]

19. Which of the following is a factor of
\[ 2x^5 - 3x^3 + 6x^2 - 5x + 6 \]

a. \( x + 1 \) 
   b. \( x - 2 \) 
   c. \( x + 2 \) 
   d. \( x - 1 \)
1. Consider the following functions.

\[
\begin{align*}
  f(x) &= 6x - 2 \\
  g(x) &= 3x \\
  h(x) &= 2^x + 3 \\
  p(x) &= 2
\end{align*}
\]

a. Determine \((f + g)(x)\)  

b. Determine \((f + h)(x)\)  

c. Determine \((g - f)(x)\)  

d. Determine \((p \cdot g)(x)\)  

e. Determine \((f \cdot g)(2)\)  

f. Determine \(\left(\frac{f}{g}\right)(x)\)

2. Given the following partial set of values of function evaluate the following.

<table>
<thead>
<tr>
<th>(x)</th>
<th>-1</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>(f(x))</td>
<td>1</td>
<td>2</td>
<td>4</td>
<td>8</td>
<td>16</td>
</tr>
<tr>
<td>(g(x))</td>
<td>-5</td>
<td>-3</td>
<td>-1</td>
<td>1</td>
<td>3</td>
</tr>
</tbody>
</table>

a. Determine \(f(1) - 2 \cdot g(2)\)  

b. Determine \((f + g)(2)\)

3. Given the following partial set of values of function evaluate the following.

a. Determine \(f(4) + 2 \cdot g(1)\)
4. Consider the following functions.

\[ f(x) = 6x - 2 \quad g(x) = 3x \quad h(x) = 2^x + 3 \quad p(x) = 2 \]

a. Determine \((f \circ h)(1)\)  
b. Determine \((g \circ f)(2)\)

c. Determine \((f \circ g)(x)\)  
d. Determine \((g \circ h)(x)\)

5. Given the following partial set of values of function evaluate the following.

<table>
<thead>
<tr>
<th>(x)</th>
<th>-1</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>(f(x))</td>
<td>1</td>
<td>2</td>
<td>4</td>
<td>8</td>
<td>16</td>
</tr>
<tr>
<td>(g(x))</td>
<td>-5</td>
<td>-3</td>
<td>-1</td>
<td>1</td>
<td>3</td>
</tr>
</tbody>
</table>

a. Determine \((f \circ g)(2)\)  
b. Determine \((g \circ f)(0)\)

6. Given the following partial set of values of function evaluate the following.

a. Determine \((f \circ g)(0)\)
7. Given the length of a rectangle can be described by the function \( f(x) = 6x - 2 \) and the width of the same rectangle can be described by \( g(x) = 2x + 1 \)

a. Determine \((f \cdot g)(x)\) and tell what it would represent as far as the rectangle is concerned.  
b. Determine an expression that represents the perimeter of the rectangle.

8. A pyramid is created from a square base where each side is 6 cm. The volume of the pyramid can be given by the function, \( V(h) = 12h \).

The function to calculate the height of the same square based pyramid given the slant height could be described by \( H(s) = \sqrt{s^2 - 9} \) where 's' is the slant height in cm.

a. Evaluate \( V(H(5)) = \)

b. Explain what \( V(H(5)) \) represents.

9. After being filled a party balloon’s volume is dependent of the temperature in the room. The volume of the balloon can be modeled by \( V(c) = 4100 + 15c \) where the volume, \( V \), is measured in cubic centimeters and the temperature, \( c \), is measured in degrees Celsius.

A function that enables you to change from degrees Celsius (C˚) to degrees Fahrenheit (F˚) is \( C(f) = \frac{5}{9}(f - 32) \).

Show the composition of function required to determine the volume of the balloon when the temperature is 98˚F.

10. A square is increased by doubling one side and decreasing the other sided by 3 units. Let \( x \) be the length of a side of the square.

Create a function that would represent a change in the area of the rectangle after the transformation.
Inverses of Functions

1. Find the inverse functions of the following.
   a. \( f(x) = 5x + 3 \)
   b. \( g(x) = \frac{3x + 1}{5} \)
   c. \( h(x) = \frac{x}{2} - 6 \)
   d. \( g(x) = 2x^3 + 6 \)

Inverse of a Function conceptually

\[ f(x) = \frac{3x - 4}{2} \]

\[ f^{-1}(x) = \]

\[ g(x) = \frac{3x + 1}{5} \]

\[ g^{-1}(x) = \]
2. Given the graph create an inverse graph and determine if the inverse is a function.

a. 

b. 

c. 

d. 

Create an inverse of the graph shown

Is the inverse a function?

CIRCLE ONE:

YES NO

M. Winking    Unit 2-5  page 39
3. Which two functions could be inverses of one another based on the partial set of values in the table?

<table>
<thead>
<tr>
<th>x</th>
<th>f(x)</th>
<th>x</th>
<th>g(x)</th>
<th>x</th>
<th>h(x)</th>
</tr>
</thead>
<tbody>
<tr>
<td>-2</td>
<td>0</td>
<td>0</td>
<td>-2</td>
<td>-2</td>
<td>-1</td>
</tr>
<tr>
<td>-1</td>
<td>2</td>
<td>2</td>
<td>-1</td>
<td>-1</td>
<td>-2</td>
</tr>
<tr>
<td>0</td>
<td>-2</td>
<td>-2</td>
<td>0</td>
<td>0</td>
<td>2</td>
</tr>
<tr>
<td>2</td>
<td>-1</td>
<td>-1</td>
<td>2</td>
<td>2</td>
<td>0</td>
</tr>
</tbody>
</table>

4. Find the inverse functions of the following using the $x \leftrightarrow y$ flip technique.

a. $g(x) = \frac{3x+1}{5}$

c. $f(x) = \frac{x+3}{x-2}; \ x \neq 2$

b. $h(x) = \frac{3}{2x+1}; \ x \neq -\frac{1}{2}$

d. $m(x) = \frac{2x-3}{3x+1}; \ x \neq -\frac{1}{3}$
5. Find the inverse functions of the following using any method:
   a. \( f(x) = 2x^2 - 3x \)               b. \( g(x) = x^2 - 4 \quad ; \quad x \geq 0 \)

6. Verify which of the following are inverses of one another by considering \( f(g(x)) \) and \( g(f(x)) \)
   a. \( f(x) = 4x \quad g(x) = \frac{x}{4} \)
   b. \( f(x) = 2x + 1 \quad g(x) = \frac{x-1}{2} \)
   c. \( f(x) = \frac{x}{2} - 3 \quad g(x) = 2x + 3 \)
   d. \( f(x) = 2x^3 - 1 \quad g(x) = \sqrt[3]{\frac{x+1}{2}} \)