An important and famous German mathematician, Carl Friedrich Gauss, is credited with first proving the **Fundamental Theorem of Algebra** which states:

> "Every polynomial equation of degree 1 or greater has at least one root in the set of complex numbers."

We can use an extension of this theorem to suggest that any polynomial of degree \( n \), must have \( n \) complex linear factors.

1. How many complex linear factors must each of the following polynomials have?
   a. \( x^4 - 3x^3 + 4x^2 - 7x + 1 \)
   b. \( 2x^3 + 3x^5 - 5x^6 - 2x + 6 \)

2. Consider the polynomial function \( f(x) \) is shown in the graph. Answer the following questions.
   a. List all of the zeros of \( f(x) \).
   b. Assuming all of the factors of the polynomial are real and the leading coefficient is 1, create a polynomial function in factored form that should describe \( f(x) \).

   \[ f(x) = \]

   c. Rewrite the polynomial function, \( f(x) \), in expanded form.

   \[ f(x) = \]

(Compare the degree, number of linear factors, and number of zeros.)

3. Consider the polynomial function \( h(x) \) is shown in the graph. Answer the following questions.
   a. Create a polynomial function in factored form for \( h(x) \), using the graph and given that \( h(x) \) has complex zeros at \( x = i \) and \( x = -i \).

   \[ h(x) = \]

   b. Rewrite the polynomial function, \( h(x) \), in expanded form.

   \[ h(x) = \]

(Compare the degree, number of linear factors, and number of zeros.)
4. Consider the polynomial function \( g(x) \) that has zeros at \( x = 3, x = -\sqrt{2}, \) and \( x = \sqrt{2} \)
   a. What is the minimum degree of the polynomial function \( g(x) \).
   
   b. Assuming all of the coefficients of the polynomial are real and the leading coefficient is 2, create the polynomial function in factored form that should describe \( g(x) \).
   
   \[
   g(x) =
   \]
   
   c. Rewrite the polynomial function, \( g(x) \), in expanded form.
   
   \[
   g(x) =
   \]

5. Consider the polynomial function \( p(x) \) that has zeros at \( x = 2, x = -2, \) and \( x = 4 \)
   a. What is the minimum degree of the polynomial function \( p(x) \).
   
   b. Assuming all of the coefficients of the polynomial are real and the function passes through the point \((1, 27)\), create an algebraic polynomial in factored form that should describe \( p(x) \).
   
   \[
   p(x) =
   \]
   
   c. Rewrite the polynomial function, \( p(x) \), in expanded form.
   
   \[
   p(x) =
   \]

6. Consider the polynomial function \( q(x) \) that has zeros at \( x = 1 \) and \( x = 3i \),
   a. What is the minimum degree of the polynomial function \( q(x) \).
   
   b. Assuming all of the coefficients of the polynomial are real and the leading coefficient is 1, create the polynomial function in factored form that should describe \( q(x) \).
   
   \[
   q(x) =
   \]
   
   c. Rewrite the polynomial function, \( q(x) \), in expanded form.
   
   \[
   q(x) =
   \]
7. Consider the polynomial function $m(x)$ is shown in the graph that has a zero of multiplicity 2. Answer the following questions.
   a. List all of the zeros of $m(x)$ and note any zeros that have a multiplicity of 2 or higher.

   b. Assuming all of the factors of the polynomial are real and the leading coefficient is 1, create a polynomial function in factored form that should describe $m(x)$.

   \[ m(x) = \]

   c. Rewrite the polynomial function, $m(x)$, in expanded form.

   \[ m(x) = \]

(Compare the degree, number of linear factors, and number of zeros.)

8. Based on the degree of the polynomial function and the graph determine how many real and how many imaginary zeros the polynomial must have. Also identify any zero’s having a higher multiplicity.

   a. \[ f(x) = x^3 - 4x^2 + x + 6 \]

   b. \[ h(x) = -x^4 + 5x^2 + 1 \]

   c. \[ g(x) = x^5 + 2x^4 - 4x^2 - 5x + 6 \]