Sec 3.6 - Polynomial Functions

Characteristics of Polynomial Functions

Name:

   A. Consider the following function (Approximate)
   B. Consider the following function. (To the nearest tenth)

   i) Local Minima: (-2, 2.8) (1, -6.3)
   ii) Local Maxima: (-1.4, 4.3)
   iii) Describe the Domain: \( \mathbb{R} \)
   iv) Describe the Range: \( y \geq -6.8 \) \( \cup \) \( y \leq 4.3 \)
   v) Describe Intervals of Increase: \( (-\infty, -1) \cup (1, \infty) \)
   vi) Describe Intervals of Decrease: \( (-2, -1) \cup (-1, 1) \)
   vii) As \( x \to -\infty \), determine \( f(x) \to -\infty \) (down)
   viii) As \( x \to \infty \), determine \( f(x) \to \infty \) (up)
   ix) Determine the \( x \)-intercept: (0, 0)
   x) Determine the \( y \)-intercept: (0, 0)

   C. Sketch a graph of \( h(x) \). (Find all answers to the nearest tenth.)

   i) Local Minima: (-1.1, -8.0) (1.9, -5.0)
   ii) Local Maxima: (-0.6, -1.4)
   iii) Describe the Domain: \( \mathbb{R} \)
   iv) Describe the Range: \( y \geq -8.0 \) \( \cup \) \( y \leq -5.0 \)
   v) Describe Intervals of Increase: \( (-1.1, 0.6) \cup (1.9, \infty) \)
   vi) Describe Intervals of Decrease: \( (-\infty, -1.1) \cup (0.6, 1.9) \)
   vii) As \( x \) increases, determine \( f(x) \to \) increases (up)
   viii) As \( x \) decreases, determine \( f(x) \to \) increases (up)
   ix) Determine the \( x \)-intercept: (-1.8, 0) (2.6, 0)
   x) Determine the \( y \)-intercept: (0, -3)
2. Determine the x-intercept and y-intercept of each of the following (use your graphing calculator for help). Round answers to the nearest hundredth when necessary.
   a. \( f(x) = x^4 + x^3 - x^2 + x - 2 \)  
   \[ y_{\text{int}} (\text{let } x=0): f(0) = 0^4 + 0^3 - 0^2 + 0 - 2 = -2 \]
   \[ y_{\text{int}} (x=0): g(0) = 2(0)^3 - 6(0)^2 + (0) + 3 = 3 \]
   \[ x_{\text{int}}(-2,0) \quad \frac{1}{2}(1,0) \]

   b. \( g(x) = 2x^3 - 6x^2 + x + 3 \)  
   \[ y_{\text{int}} (x=0): g(0) = 2(0)^3 - 6(0)^2 + (0) + 3 = 3 \]
   \[ x_{\text{intercept(s)}}: (-0.58, 0) \quad (1,0) \quad (0.58, 0) \]

3. Based on the following partial set of table values of a polynomial function, determine between which two x-values you believe a zero may have occurred.
   a. 
<table>
<thead>
<tr>
<th>x</th>
<th>y</th>
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<tbody>
<tr>
<td>-2</td>
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<td>-1</td>
<td>0</td>
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<td>-5</td>
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<td>5</td>
<td>2</td>
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<td>6</td>
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</tbody>
</table>
   
   Zeros occur at: \( x=-1, x=2 \)  
   and between \( x=5 \) \& \( x=0 \)

4. Based on the following partial set of table values of a polynomial function, determine between which two values you believe a local maximum or local minimum may have occurred.
   a. 
<table>
<thead>
<tr>
<th>x</th>
<th>y</th>
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<tbody>
<tr>
<td>-2</td>
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</tbody>
</table>
   
   Local max between \( x=0 \) \& \( x=1 \)  
   Local min between \( x=4 \) \& \( x=5 \)  
   Local maximum between \( x=1 \) \& \( x=3 \)

5. The following are graphs are of polynomial functions. Determine which of the following have an EVEN or ODD degree and whether the leading coefficient is POSITIVE or NEGATIVE.
   a. \( \text{Even degree functions have the same end behavior for } x \to \infty \) \& \( x \to -\infty \) \( \text{i.e. both up or both down} \)
   b. \( \text{Odd degree functions have opposite end behavior at each end for } x \to \infty \) \& \( x \to -\infty \) \( \text{i.e. one side goes up and the other down} \)
   c. \( \text{Degree odd lead coeff. negative down} \)
   d. \( \text{Degree even lead coeff. positive up} \)
7. Polynomial Functions in context.
   a. A baseball is struck such that its position on a plane perpendicular to home plate can be described by the equation 
   \[ y = -\frac{1}{250}(x - 175)^2 + 121 \], where \( x \) is the horizontal distance from the plate in feet and \( y \) is the vertical distance also in feet.

   Determine the position of the ball when it is at its highest point.
   \[ y = a(x-h)^2 + k \]
   \( (175, 121) \)
   The ball reaches a maximum height when it is a horizontal distance of 175 feet away from home plate and it is 121 feet high.

   b. A company’s net worth for the first 6 years after it opened can be modeled by the polynomial function:
   \[ p(t) = t^4 - 6t^2 + 8t \], where \( t \) is measure in years after the business opened and \( p(t) \) represent the company’s net worth in millions.

   During what years did the company have a negative net worth?

   c. Many businesses know that most consumers associate price with quality (i.e. if it costs more it must be of a higher quality). An actuary at a cosmetics manufacture determined that the profit made the price charged for lipstick at a particular retail outlet could be modeled by:
   \[ V(p) = -0.5p^3 + 12p^2 - 17p, \]
   where \( p \) is the price in dollars and \( V(p) \) is the amount earned by the retail store selling just the lipstick in a month.

   How much should the store charge to maximize its profit?

   d. A new hybrid car’s fuel economy (MPG) depends on how fast the engine is running in RPM’s. The following polynomial function model is reasonable accurate from 20mph to 80 mph.
   \[ F(s) = -0.0002s^4 + 0.0356s^3 - 2.3022s^2 + 68.832s - 588, \]
   where \( s \) is the speed in miles per hour (MPH) and \( F(s) \) is the fuel economy in MPG.

   What speeds would between 20 and 60 mph would maximize fuel economy?