When a weighted average is applied to a set of numbers, more importance (weight) is placed on some components of the set. Your final average in this class I probably an example of a weighted average.

Consider two grading systems

<table>
<thead>
<tr>
<th>Mr. Tats Grade Weighting</th>
<th>Mrs. Etercsid Grade Weighting</th>
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</thead>
<tbody>
<tr>
<td>Homework Average</td>
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<tr>
<td>25%</td>
<td>15%</td>
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<tr>
<td>Class Participation</td>
<td>Class Participation</td>
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<tr>
<td>10%</td>
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<tr>
<td>Test Average</td>
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<tr>
<td>40%</td>
<td>60%</td>
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<tr>
<td>Final Exam Grade</td>
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<td>25%</td>
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1. If you had the following grade shown below, determine what your grade would be with each teacher.

   MR. TATS AVE = 0.25(90) + 0.10(95) + 0.40(84) + 0.25(68) = 82.6

   MRS. E AVE = 0.15(90) + 0.10(95) + 0.60(84) + 0.15(68) = 83.6

2. Use the following information to find your course average in Mr. Tats class:
   - Homework {83, 92, 95, 90} \[\frac{83+92+95+90}{4} = 90\]
   - Class Participation {90, 100} \[\frac{90+100}{2} = 95\]
   - Test Grades {90, 78, 84,88} \[\frac{90+78+84+88}{4} = 85\]
   - Final Exam Grade {84}

   \[0.25(90) + 0.10(95) + 0.40(85) + 0.25(84) = 87\]

3. If Mr. Tats allowed you to retake your final exam, what score would you have to get on the final exam to make at least a 90% in the class?

   \[0.25(90) + 0.10(95) + 0.40(85) + 0.25x = 90\]
   \[0.25x = 24\]
   \[x = 96\]
Mr. Tats Grade Weighting

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Mrs. Etercsid Grade Weighting

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4. The following shows a student’s grade in Ms. Etercsid’s class. Determine the minimum grade need on the final exam to make at least a 70% in the class.
   • Homework {70, 82, 64} \[\frac{70+82+64}{3} = 72\]
   • Class Participation {70, 80} \[\frac{70+80}{2} = 75\]
   
   \[0.15(72) + 0.10(75) + 0.50(66) + 0.15x = 70\]
   \[57.9 + 7.5 + 33 + 0.15x = 70\]
   \[57.9 + 10.5 + 0.15x = 70\]
   \[68.4 + 0.15x = 70\]
   \[0.15x = 1.6\]
   \[x = 10.6\]

Other weighted averages

5. A new cars salesperson, sold an average of $28,000 per day for the first 5 days she worked. The next 3 days she worked, she sold an average of $42,000. How much did she sell on average per day over the entire 8 days?
   
   \[\frac{28000+28000+28000+28000+42000+42000+42000+42000}{8} = 33250\]

6. Kelly works two jobs. She usually works at a retail store for 7 hours during the day and is paid $12 per hour by the store. In the evening she works at a restaurant as a manager for 5 hours and is paid $20 per hour. How much does she make on average per hour for the day?
   
   \[\frac{7(12) + 5(20)}{12} = \$15.33\text{ PER HOUR}\]

7. James worked for 12 consecutive days, earning an average wage of $160 per day as a contractor. During the first 6 days, James averaged $140 per day and his average wage for the next 5 days was $175. How much did he earn on the last day?
   
   \[\frac{6(140) + 5(175) + x}{12} = 160\]
   \[1715 + x = 1920\]
   \[x = 205\]
**Slugging Percentage**

In baseball there are many common statistics used batting average and slugging percentage. Batting average is just simply the percentage of the number of “at bats” for which the player gets a hit. If a player has 20 hits in 50 at bats then their batting average would be, \( \frac{20}{50} = 0.400 \). Their slugging percentage however, weights the type of hit where doubles count double, triples count triple, and homeruns count quadruple and would be given by the formula:

\[
\text{Slugging Percentage} = \frac{(\text{singles}) + 2 \cdot (\text{doubles}) + 3 \cdot (\text{triples}) + 4 \cdot (\text{home runs})}{\text{(at bats)}}
\]

8. Calculate Freddie’s Slugging Percentage if he has 42 singles, 12 doubles, 4 triples, and 14 home runs in 230 at bats.

\[
\text{SP} = \frac{(42) + 2(12) + 3(4) + 4(14)}{230} \approx 0.583
\]

9. Can you use the above information to determine Freddie’s current batting average?

\[
\text{BattAvg} = \frac{42 + 12 + 4 + 14}{230} \approx 0.313
\]

10. If Andrelton has a slugging percentage of 0.450 and has 12 doubles, 3 triples, and 8 homeruns in 200 at bats then how many singles must he have?

\[
\frac{x + 2(12) + 3(3) + 4(8)}{200} = 0.450
\]

\[
x + 2(12) + 3(3) + 4(8) = 90
\]

\[
x + 65 = 90
\]

\[
x = 25
\]

11. Can you determine Andrelton’s current batting average?

\[
\frac{25 + 12 + 3 + 8}{200} = 0.240
\]

\[
\frac{(25 + 12 + 3 + 8)}{200} \cdot 24
\]
Universal Product Codes (UPCs), typically in the form of barcodes

Identification numbers are present everywhere in society. Today’s identification numbers are more sophisticated than those introduced years earlier (for example, Social Security numbers). Today’s numbers have a check digit to partially ensure that they have been correctly scanned or entered into a computer.

Universal Product Codes (UPCs), typically in the form of barcodes, identify retail products.

The 12-digit UPC barcode consists of three parts:
- manufacturer number,
- product number, and
- check digit.

For example, the manufacturer number for the Dr. Pepper Company is 078000 and appears in the first six digits of all of the company’s product UPC barcodes. GS1, formerly the Uniform Code Council, issues a company this six-digit number. Every item sold by a company requires a different five-digit product number. This includes specific products, their different sizes, their array of colors, their variety of flavors, and other distinguishing features. The last number is the check digit, which guards against entry errors and fraud. The check digit in a UPC number (that is, the twelfth digit) is determined in the following manner:
- Multiply the first digit by 3.
- Add the second digit.
- Multiply the third digit by 3.
- Add the fourth digit.
- Continue this alternating process for the Digits 5 to 12.

The check digit is chosen so that the calculation described previously totals a number whose final digit is 0. In the UPC number $a_1a_2a_3a_4a_5a_6a_7a_8a_9a_10a_11d$, the check digit is $d$, for which the sum

$$3a_1 + a_2 + 3a_3 + a_4 + 3a_5 + a_6 + 3a_7 + a_8 + 3a_9 + a_{10} + 3a_{11} + d$$

ends in 0. In this weighted sum, the weights are: \{3, 1, 3, 1, 3, 1, 3, 1, 3, 1, 3, 1\}.

When entering a code number, the single-digit error is most common (for example, keying in 8 instead of 3). Another common error is the transposition error, where the order of two adjacent digits is reversed (for example, writing 83 instead of 38). Systems have been established to detect and correct (when possible) these and other errors almost immediately.

1. Show that 0-58200-48826-5 is a valid UPC number.

   $$3(0+8+0+0+4+8+0) + (5+2+0+0+8+2) + 5 = \text{VALID CHECK DIGIT}$$

2. Show that 0-52200-48826-5 is an invalid UPC number.

   $$3(0+2+0+4+8+0) + (5+2+0+0+8+2) + 5 = \text{NOT A VALID CHECK DIGIT}$$

3. Determine the check digit ($d$) for the UPC number 3-81370-09213-d.

   $$3(3+1+7+0+2+2) + (8+3+0+9+1) + d = 109 + d = \text{MUST BE A MULTIPLE OF TEN AND d MUST BE AN INTEGER FROM 0-9}$$
Credit Cards

Identification numbers are present everywhere in society. Today’s identification numbers are more sophisticated than those introduced years earlier (for example, Social Security numbers). Today’s numbers have a check digit to partially ensure that they have been correctly scanned or entered into a computer.

Credit cards have 16-digit numbers, of which the first 15 digits identify the credit card and the sixteenth digit is the check digit. The following figure shows the significance of the digits:

MII stands for major industry identifier; 4 and 5 indicate “Banking and Financial.” VISA cards begin with 4 and MasterCard cards with 5.

1. MasterCard numbers begin with 51, 52, 53, 54, or 55. What is the maximum number of credit cards that MasterCard can issue?

A check digit is used to help validate credit card numbers. The credit card companies use the Codabar method to determine the check digit. This method consists of the following steps:

- Add the digits in the odd-numbered positions and double this total.
- Add the number of odd-position digits that are more than 4 to the total.
- Add the even-position digits.
- Choose a check digit that makes this calculation total a number whose final digit is 0.

Libraries, shipping/receiving companies, and blood banks also use the Codabar method.

2. Show that the check digit (d) for the VISA card 4162 0012 3456 789d is 3.

3. What is the check digit (d) for the MasterCard number 5424 9813 2720 008d?

4. Show that 4128 0012 4389 0110 is an invalid VISA credit card number.