Sec 5.7 - Exponential & Logarithmic Functions
(Solving Exponential Equations)

1. Solve the following basic exponential equations by rewriting each side using the same base.
   a. \(3^{x-1} = 81\)
      \[3^{x-1} = 3^4\]
      \[x - 1 = 4\]
      \[x = 5\]
   b. \(2^{2x-3} = 128\)
      \[2^{2x-3} = 2^7\]
      \[2x - 3 = 7\]
      \[2x = 10\]
      \[x = 5\]
   c. \(2^{x-1} = 4^3\)
      \[2^{x-1} = (2^2)^3\]
      \[2x - 1 = 6\]
      \[2x = 7\]
      \[x = 3.5\]

2. Solve the following basic exponential equation by rewriting each as logarithmic equation and approximating the value of \(x\).
   a. \(4^x = 102\)
      \[\log_4(4^x) = \log_4(102)\]
      \[x = \log_4(102)\]
      \[x \approx 3.236\]
   b. \(5^{2x-3} = 1953125\)
      \[\log_5(5^{2x-3}) = \log_5(1953125)\]
      \[2x - 3 = 9\]
      \[2x = 12\]
      \[x = 6\]
   c. \(e^{2x} = 78\)
      \[\log_e(e^{2x}) = \log_e(78)\]
      \[2x = \ln(78)\]
      \[x = \frac{\ln(78)}{2} \approx 2.178\]

3. Solve the following exponential equation by rewriting each as logarithmic equation and approximating the value of \(x\).
   a. \(6^{2x} - 8 = 112\)
      \[6^{2x} = 120\]
      \[\log_6(6^{2x}) = \log_6(120)\]
      \[2x = \log_6(120)\]
      \[x \approx 1.336\]
   b. \(2 \cdot 3^{x-3} + 1 = 39367\)
      \[\log_2(2 \cdot 3^{x-3}) + 1 = \log_2(39367)\]
      \[\frac{2 \cdot 3^{x-3}}{2} + 1 = \frac{39367}{2}\]
      \[3^{x-3} = 19683\]
      \[\log_3(3^{x-3}) = \log_3(19683)\]
      \[x - 3 = 9\]
      \[x = 12\]
   c. \(3e^{2x} + 2 = 92\)
      \[\frac{3e^{2x}}{3} = \frac{90}{3}\]
      \[e^{2x} = 30\]
      \[\log_e(e^{2x}) = \log_e(30)\]
      \[2x = \log_e(30)\]
      \[x \approx 1.701\]
   d. \(2 \cdot 4^{2x+1} + 40 = 8\)
      \[\log_2(2 \cdot 4^{2x+1}) + 40 = \log_2(8)\]
      \[\frac{2 \cdot 4^{2x+1}}{2} + 40 = \frac{8}{2}\]
      \[4^{2x+1} = -16\]
      \[\log_4(4^{2x+1}) = \log_4(-16)\]
      \[2x + 1 = \log_4(-16)\]
      \[2x + 1 \approx 2 + 2.266i\]
      \[x \approx 0.5 + 1.133i\]
4. Solve the following exponential inequalities.

a. \(3^x - 7 > 20\)
   \[3^x > 27\]
   \[\log_3(3^x) > \log_3(27)\]
   \[x > 3 \text{ : SET NOTATION}\]
   \[(3,\infty) \text{ : INTERVAL NOTATION}\]

b. \(5^{x-3} \geq -40\)
   ANY REAL VALUE FOR \(x\) WILL RESULT IN THE LEFT SIDE BEING POSITIVE.
   SO, ANY REAL VALUE FOR \(x\) SATISFIES THE STATEMENT.
   SET: \(x = \text{ALL REALS (R)}\)
   INTERVAL: \(x: (-\infty, \infty)\)

c. \(2e^{2x} + 1 < 33\)
   \[2e^{2x} < 32\]
   \[\frac{2e^{2x}}{2} < \frac{32}{2}\]
   \[e^{2x} < 16\]
   \[\log_e(e^{2x}) < \log_e(16)\]
   \[2x < \ln(16)\]
   \[x < \frac{\ln(16)}{2} \text{ : SET NOTATION}\]
   \[x: \left(-\infty, \frac{\ln(16)}{2}\right) \text{ : INTERVAL NOTATION}\]

d. \(5 \cdot 4^{2x+1} - 2 \geq 243\)
   \[\frac{5 \cdot 4^{2x+1}}{2} \geq \frac{243}{2}\]
   \[\frac{5 \cdot 4^{2x+1}}{2} \geq 121.5\]
   \[\log_4\left(\frac{5 \cdot 4^{2x+1}}{2}\right) \geq \log_4(121.5)\]
   \[\log_4\left(\frac{5 \cdot 4^{2x}}{2}\right) \geq \log_4(121.5)\]
   \[\log_4\left(\frac{5 \cdot 2^{2x+1}}{2}\right) \geq \log_4(121.5)\]
   \[\log_4\left(\frac{5 \cdot 2^{2x}}{2}\right) \geq \log_4(121.5)\]
   \[\log_4(2^{2x}) \geq \log_4(121.5)\]
   \[2x \geq 10.3\]
   \[x \geq 5.15\]
   \[\text{SET: } x \geq 5.15 \text{ : INTERVAL NOTATION}\]

5. Solve the following applications

a. Create an equation that represents the value \(P\) of an investment \(t\) years after the initial investment. The initial investment was $3200 and increases by 12% each year (compounded annually). This would suggest that the account value could be modeled by \(P = 3200 \cdot (1.12)^t\). Determine how many years it should take for the investment to double in value.

\[
\frac{6400}{3200} = 1.12^t
\]
\[2 = 1.12^t\]
\[\log_{1.12}(2) = t\]
\[
\log(2) \cdot \log(1.12)
\]
\[6.116255374
\]
\[t \approx 6\ \text{YEARS}
\]

b. There are 15 virus particles known as a virion in a host and the number of virions doubles every hour and continues this model for the first 28 hours. The number of \(N\) virion after \(t\) hours can be found using the formula \(N = 15 \cdot (2^t)\). How long will it take approximately for there to be 4,500,000 virions living in the host?

\[
\frac{4,500,000}{15} = 15 \cdot (2^t)
\]
\[300,000 = 2^t\]
\[\log_2(300,000) = \log_2(2^t)\]
\[
\log(300000) / \log(2)
\]
\[18.19460298\]
\[t \approx 18.2 \\text{HOURS}\]

c. There are initially 8 frogs living in a pond in the back of a farm. The number frogs can described by the formula \(P = 8 \cdot e^{0.2t}\). How many years will it take for the population of frogs to grow to 100 if the model continues?

\[
\frac{100}{8} = e^{0.2t}
\]
\[12.5 = e^{0.2t}\]
\[\log_e(12.5) = \log_e(e^{0.2t})\]
\[
\ln(12.5) / 0.2
\]
\[12.62864322\]
\[t \approx 12.6 \text{ YEARS}\]