Name:

1. James is borrowing $2000 from his employer and will pay it back at the end of 3 years. The employer lending the money asks that James pay **simple interest** of 3% annually. Using the simple interest formula, $I = P \cdot R \cdot T$, determine how much interest James will have to payback in addition to the $2000 principal amount.

$I = P \cdot R \cdot T$ — P: PRINCIPLE, R: RATE AS DECIMAL, T: TIME IN YEARS

$I = (2000)(.03)(3)$

$= \$180$

PAYMENT: $2180

2. Ryan is investing $9000 in a CD at a bank. If the bank uses **simple interest** and the bank pays 3.1% annually, how much will the CD be worth in total at the end of 5 years when the CD matures?

$I = (9000)(.031)(5)$

$I = 1395$

CD IS WORTH $= 9000 + 1395$

$= \$10,395$

3. Jim put his $15,000 into a high yields savings account that pays 2.8% annually.

   a. The account is compounded annually. If the bank uses a **compound interest** formula, how much will the account be worth in 5 years if left untouched? Use the compound interest formula, $P = P_o\left(1 + \frac{r}{n}\right)^{nt}$

   P_o: INITIAL PRINCIPLE, r: RATE AS DECIMAL, n: NUMBER OF COMPOUNDS PER YEAR, t: TIME

   $= (15000)\left(1 + \frac{.028}{1}\right)^{(1 \cdot 5)} = \$17,220.94$

   (15000)(1+.028/1)^(1*5)
   17220.93916

   b. Jim saw that other banks offered the same rates but compounded the interest more often. Consider if he still put $15,000 into a savings account for 5 years that provided 2.8% annually but compounded it in each of the following ways (fill out the table):

   (15000)(1+.028/2)^(2*5)
   17237.36227

Compounded	Work:	Value in 5 Years
Semi-Annually $n = 2$	$P = (15000)\left(1 + \frac{.028}{2}\right)^{(2 \cdot 5)} =$	$17237.36
Monthly $n = 12$	$P = (15000)\left(1 + \frac{.028}{12}\right)^{(12 \cdot 5)} =$ (15000)(1+.028/12)^(12*5) 17251.29342	$17,251.29
Quarterly $n = 4$	$P = (15000)\left(1 + \frac{.028}{4}\right)^{(4 \cdot 5)} =$ (15000)(1+.028/4)^(4*5) 17245.69377	$17,245.69
Weekly $n = 52$	$P = (15000)\left(1 + \frac{.028}{52}\right)^{(52 \cdot 5)} =$ (15000)(1+.028/52)^(52*5) 17253.45688	$17,253.46
Daily $n = 365$	$P = (15000)\left(1 + \frac{.028}{365}\right)^{(365 \cdot 5)} =$ (15000)(1+.028/365)^(365*5) 17254.01434	$17,254.01
Continuously	$P = P_o e^{rt} = (15000)e^{(.028 * 5)}$ (15000)e^(.028*5) 17254.10698	$17,254.11

Why does your investment increase more as your compounds per year increase?

(15000)(1+.028/1000)^(1000*5)
17254.07317

When making the jump to continuously compounding the transcendental number 'e' appears. Can you determine a definition for 'e' given:

$$\lim_{n\to\infty} P_o\left(1+\frac{r}{n}\right)^{nt} = P_o e^{rt}$$

Simple Interest: $P = P_o + P_o rt$	P = Principle	*Annually: n = 1*
Compound Interest: $P = P_o\left(1+\frac{r}{n}\right)^{nt}$	P_0 = Initial Principle	*Semi-Annually: n = 2*
Continuous Interest: $P = P_o e^{rt}$	r = rate as a decimal	*Monthly: n = 12*
	t = number of years	*Weekly: n = 52*
	n =compounds per year	*Daily: n = 365*

4. Determine which investment is **best**. Evan plans to invest \$10000 for 8 years.
   a. Better Banks offers an 8 year CD at an annual rate of 5% using **simple interest**.

$$P = (10000) + (10000)(.05)(8) = \$14{,}000$$

(10000)+(10000)(.05)(8)
14000

   b. America's Bank offers an 8 year CD at annual rate of 4.7% using **monthly compound interest**.

$$P = (10000)\left(1+\frac{.047}{12}\right)^{(12\cdot 8)} = \$14{,}553.78$$

(10000)(1+.047/12)^(12*8)
14553.77888

   c. Union Bank offers an 8 year CD at an annual rate of 4.5% using **continuous compounding**.

$$P = (10000)e^{(.045\cdot 8)} = \$14333.29$$

(10000)e^(.045*8)
14333.29415

5. Marissa wants to invest \$4000 in a retirement fund that guarantees a return of 9% annually using **simple interest**. How many years and months will it take for her investment to double?

$$P = P_o + P_o r t$$
$$8000 = (4000) + (4000)(.09)t$$
$$-4000 \quad -4000$$
$$4000 = 360t$$
$$\frac{4000}{360} = \frac{360t}{360}$$
$$11.\overline{1}\ \text{YEARS} = t$$

6. Jeff wants to invest \$4000 in a retirement fund that guarantees a return of 8% annually using **continuously compounded interest**. How many years and months will it take for his investment to double?

$$P = P_o e^{rt}$$
$$\frac{8000}{4000} = \frac{4000e^{.08t}}{4000}$$
$$2 = e^{.08t}$$
$$\ln(2) = \ln(e^{.08t})$$
$$\frac{\ln(2)}{.08} = \frac{.08t}{.08}$$
$$8.66\text{ yrs} = t$$

ln(2)/.08
8.664339757

7. Ten years ago, Josh put money into an account paying 5.5% **compounded continuously**. If the account has \$12,000 now, how much money did he deposit?

$$P = P_o e^{rt}$$
$$12000 = P_o e^{(.055\cdot 10)}$$
$$12000 = P_o \cdot 1.733$$
$$\frac{12000}{1.733} = \frac{1.733P_o}{1.733}$$
$$\boxed{6924.41 \approx P_o}$$

e^(.055*10)
1.733253018

e^(.055*10)
1.733253018
12000/1.733
6924.40854

Ever worry about being ripped off at a car dealership? Are you sure the finance office is calculating your payment correctly or did they add something without your knowledge? With the formula for determining monthly payments using simple interest on an amortized loan is as follows:

First you need to know the givens:

P = Principle (Amount of the Loan) n = number of payments per year

r = Finance rate as a decimal t = number of years

$$Monthly\ \ Payment = \frac{P \cdot \frac{r}{n} \cdot \left(1+\frac{r}{n}\right)^{nt}}{\left(1+\frac{r}{n}\right)^{nt} - 1}$$

Let's try one. So you want to buy a car? Consider looking at the new 2015 Honda Civic Si. For now we will just look at the financing and not consider the huge insurance price tag.

MSRP	**$24,950**
Dealer Destination	**$595**
Under Body Spoiler Side	**$599**
Under Body Spoiler Front	**$356**
Under Body Spoiler Rear	**$356**
Fog Lights	**$450**
Moon roof Visor	**$138**
Floor Mats	**$65**
Wheels: 18" 5-Spoke Aluminum	**$1820**
	$29,329

Maybe if you're lucky, you can talk them down to $27,000. Then, with all of that hard earned money from TARGET you might have saved up $3000 for a down payment? So, we will have to finance ***$24,000***. Let's see if you could afford the payments on this car over ***5 years*** and with a ***6.5% APR***.

Convert 6.5% to a decimal ________0.065______

Since $\frac{r}{n}$ is used so much in the formula lets go ahead and determine that number.

$$\frac{r}{n} = \frac{0.065}{12} \approx 0.0054167$$

Now, it is just a matter of substituting everything correctly and arithmetic.

$$M.P. = \frac{P \cdot \frac{r}{n} \cdot \left(1+\frac{r}{n}\right)^{nt}}{\left(1+\frac{r}{n}\right)^{nt} - 1} = \frac{\overbrace{(\$24000)\cdot(0.005417)}\cdot(1+0.0054167)^{(12)(5)}}{(1+0.005417)^{(12)(5)} - 1} = \frac{(\$130.008)\cdot(1.005417)^{60}}{(1.005417)^{60} - 1} = \frac{\$179.78089}{0.382817}$$

$$= \boxed{\$469.63}$$

I don't think SO!! And that doesn't even include your monthly insurance.

A little side note: Do you realize you actually end up paying $469.63 × 60 months = $28,177.80. Remember we only borrowed $24,000. The bank makes over $4000 dollars just for loaning you the money and this is with a low APR.

8. On your own, determine the monthly payment for a used car and lets say you need to borrow just **$7,500** from the bank with an annual percentage rate (APR) of **9.5%** (Used car loan rates are always higher!). Usually, the longest you can have a loan on a used car is 4 years. So, lets say we will pay the loan off over a term of **4** years and make payments on a **monthly** basis.

$$Monthly\ \ Payment = \frac{P \cdot \frac{r}{n} \cdot \left(1+\frac{r}{n}\right)^{nt}}{\left(1+\frac{r}{n}\right)^{nt} - 1}$$

$P = 7500$ $r = .095$

$n = 12$ $t = 4$

$\frac{r}{n} = \frac{.095}{12} \approx .00791\overline{6}$

$$MP = \frac{(7500)(.0079167)(1+.0079167)^{(12 \cdot 4)}}{(1+.0079167)^{(12\cdot 4)} - 1} = \frac{86.6938}{.460101} \approx \boxed{\$188.42}$$

```
(7500)(.0079167)
(1+.0079167)^(12
*4)
           86.69383599
(1+.0079167)^(12
*4)-1
          .4601005637

86.6938/.460101
         188.4234114
```

- Then, determine the amount actually paid to the bank and the total amount of interest.

$(188.42 * 12 * 4) = \$9044.16$

```
188.42*12*4
            9044.16
```

INT = 9044.16 − 7500 = $1544.16

```
9044.16-7500
          1544.16
```

9. House payments work the exact same way except banks usually give you 15 to 30 years to pay it off. What would your monthly payment be on a house if you took a loan out for $85,000 with an APR of 7.1% and you were going to pay it back over a term of 30 years on a monthly basis?
(Also, with houses you can expect to pay another $100 - $300 per month in taxes and insurance depending on where you live and the price of the house.)

```
(85000)(.0059167
)(1+.0059167)^(1
2*30)
      4205.571791
(1+.0059167)^(12
*30)-1
       7.36231602
```

$$Monthly\ \ Payment = \frac{P \cdot \frac{r}{n} \cdot \left(1+\frac{r}{n}\right)^{nt}}{\left(1+\frac{r}{n}\right)^{nt} - 1}$$

$P = 85000$ $r = .071$

$n = 12$ $t = 30$

$\frac{r}{n} = \frac{.071}{12} \approx .0059167$

$$MP = \frac{(85000)(.0059167)(1+.0059167)^{(12\cdot 30)}}{(1+.0059167)^{(12 \cdot 30)} - 1} \approx \frac{4205.57179}{7.362316} = \$571.23$$

```
4205.57179/7.362
316
     571.2294596
```

- Then, determine the amount actually paid to the bank and the total amount of interest.

$571.23 \times 12 \times 30 = \$205,642.80$

```
571.23*12*30
      205642.8
```

INT = 205,642.80 − 85,000

```
205642.80-85000
       120642.8
```

= $120,642.80